Multiple Linear Regression Viewpoints

A Publication sponsored by the American Educational Research Association's Special Interest Group on Multiple Linear Regression: The General Linear Model

MLRV

Volume 27 • Number 2 • Fall 2001

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Multiple Linear Regression Viewpoints (ISSN 0195-7171) is published by the AERA Special Interest Group on Multiple Linear Regression: General Linear Model through the **University of Alabama at Birmingham**.

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A Note from the Editor of *MLRV*

T. Mark Beasley
University of Alabama at Birmingham

I would like to thank the members of the Multiple Linear Regression – General Linear Model Special Interest Group (MLR/GLM SIG) for affording me this opportunity to be the Editor of *Multiple Linear Regression Viewpoints* (*MLRV*). It is a great honor to have my name follow the likes of Isadore Newman, John Pohlmann, Keith McNeil, and Randall Schumacker. I am please to name Robin K. Henson of the University of North Texas as my Associate Editor. I believe he will be a great help.

Many changes have come along with this editorship. Namely, since being elected Editor of *MLRV*, I have taken a faculty position in the Department of Biostatistics at the University of Alabama at Birmingham (UAB). After eight years as an Assistant/Associate Professor teaching service courses in Statistics and Research Methods at St. John's University in New York, I felt it was time for a change and a new challenge. UAB presented me great prospects for research along with the opportunity to be closer to my family in my home state of Tennessee. I believe that the opportunities at UAB will help me advance my career as a statistician, but I also feel that the resources at UAB should help me to advance *MLRV* as a publication.

Several years ago, the members of the SIG officially changed the name to the Multiple Linear Regression – General Linear Model SIG. The impetus for this change was to incorporate a wider variety of topics for (a) proposals to the SIG for the American Educational Research Association (AERA) convention and (b) submissions to *MLRV*. In the 1990's, the SIG and *MLRV* recognized the linear model basis of Hierarchical Linear Models (HLMs) and Structural Equation Models (SEMs) and started including them as acceptable subject matter. To this extent, I propose further expansion to include *Generalized Linear Models*. Outside of the fact that it does not even require a change in acronym and we can still use the same monogrammed towels at the AERA convention, *Generalized Linear Models* subsume all the topics currently covered in *MLRV*.

For those of you who are not familiar with the topic, *Generalized Linear Models* are a broad class of models that include regression models for continuous dependent variables, alternative models for continuous dependent variables that do not assume normality or homoscedasticity of the residuals, and also models for discrete dependent variables (e.g., dichotomies, counts). Therefore, the multiple regression and ANOVA models typically covered in MLRV are special cases of *Generalized Linear Models*.

There are three basic components to *Generalized Linear Models*:

1. The *systematic component* specifies the explanatory variables used as the predictors, X. This is no different than the more familiar General Linear Models. The systematic component may include fixed effects, random effects, or may involve a mixed model as is common in longitudinal data and nested data structures (e.g., HLMs).

2. The *random component* identifies the dependent variable, Y, and specifies a probability distribution for the residuals. The normal distribution is but one of many known probability distributions that can be specified.

3. The *link function* specifies a function of the expected value of Y, E(Y) or $\hat{Y}$, to be predicted by the X variables. This component *links* the systematic and random components by specifying the connection between the X predictor variables and the predicted value of Y. That is, it specifies the predicted value of Y as a function of the X variables through an equation of linear form:

$$f(\hat{Y}) = \beta_0 + \beta_1 X_1 + \beta_1 X_1 + \ldots + \beta_k X_k \ .$$

For the familiar Ordinary Least Squares (OLS) linear regression model with a continuous Y, there is an *identity link function* (i.e., the predicted values are not transformed in any way) and the random error component (i.e., residuals) is assumed to have a normal distribution. However, in the closely related logistic regression model, Y is dichotomous and therefore has a binomial distribution. Thus, logistic regression is a *Generalized Linear Model* that specifies a *logit link function* that transforms $\hat{Y}$ to be a linear function of the X variables,

$$f(\hat{Y}) = \text{logit}(\hat{Y}) = \log\left(\frac{\hat{Y}}{1-\hat{Y}}\right),$$

and identifies a binomial distribution for the random error component.

For continuous dependent variables, it is possible to specify an error distribution other than the normal. For example, when the residuals display fan-shaped heteroscedasticity, it is possible that the variance of the residuals increases as a systematic function of $\hat{Y}$. One approach to this analytic problem is to transform the data in order to remove the heteroscedasticity. This approach is problematic if the researcher is interested in prediction and the variable "loses its meaning" when transformed. In an experimental design context, it is possible that a treatment may increase average level and reduce variability. Instead of transforming this "nuisance" into submission, a *Generalized Linear Model* can parameterize and model this interesting phenomenon. However, identifying the "correct" error distribution can be arduous, and therefore, nonparametric methods are still viable approaches.

Thus, the investigating the merits and statistical properties of these *Generalized Linear Models* relative to other procedures should provide a wealth of research opportunities for statisticians who conduct simulation studies. For example, if the simulation researcher creates a situation where the random error distribution is radically skewed, (s)he could compare *Generalized Linear Models*, with a variety of specified random components, to OLS regression and nonparametric methods. For applied statisticians and data analysts, manuscripts elaborating (a) how *Generalized Linear Models* relate to more familiar data analytic techniques or (b) how *Generalized Linear Models* can be applied to answer educational research questions are welcomed.

To my recollection, there has been only one paper specifically concerning *Generalized Linear Models* presented in the MLR/GLM SIG at AERA. I hope to see more and see such submissions to *MLRV*. This does not preclude other submissions, I simply want to expand the horizon for *MLRV*. This inclusion of *Generalized Linear Models* will not change the face of *MLRV* but hopefully enlarge the number of subscriptions and submissions.

Again, I thank the MLR/GLM SIG members for this wonderful opportunity and hope that I serve the editorship well.

Sincerely,

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An Empirical Investigation of Four Tests for Interaction in the Context of Factorial Analysis of Covariance

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The Type I error and power properties of the parametric *F* test and three nonparametric competitors were compared in terms of a 3×4 factorial analysis of covariance layout. The focus of the study was on the test for interaction either in the presence and/or absence of main effects. A variety of conditional distributions, sample sizes, levels of variate and covariate correlation, and treatment effect sizes were investigated. The Puri and Sen (1969a) test had ultra-conservative Type I error rates and power losses when main effect(s) were present. The adjusted rank transform (Blair & Sawilowsky, 1990; Salter & Fawcett; 1993) had liberal Type I error rates when sampling was from moderate to extremely skewed distributions. The Hettmansperger (1984) chi-square test displayed acceptable Type I error rates for all distributions considered when sample sizes were ten or twenty. It is suggested that the Hettmansperger (1984) test be considered as an alternative to the parametric *F* test provided sample sizes are relatively equal and at least as large as ten.

The rank transform (RT) procedure was recommended as an alternative to the parametric procedure in multiple regression (Iman & Conover, 1979) and factorial analysis of covariance (Conover & Iman, 1981, 1982) when the assumption of population normality was violated. The steps for hypotheses testing using the RT consists of (a) replacing the raw scores with their respective rank order, (b) conducting the classical normal theory tests on the ranks, and (c) referring to the usual tables of percentage points.

Unfortunately, the parametric *F* test is not invariant with respect to monotone transformations (such as the RT). More specifically, the nonlinear nature of the RT may add (remove) interactions when such interactions were absent (present) in the original raw scores. For example, and contrary to the suggestions above, it has been demonstrated that the RT fails as a viable alternative to the parametric procedure with respect to tests for (a) interaction in factorial ANOVA (Blair, Sawilowsky, & Higgins, 1987; Thompson, 1991; 1993), (b) parallelism and interaction in factorial ANCOVA (Headrick, 1997; Headrick & Sawilowsky, 2000), and (c) additive and nonadditive models in multiple regression (Headrick & Rotou, 2000).

However, nonparametric tests can be substantially more powerful than the parametric *t* or *F* tests when the assumption of normality is violated. For example, the Mann-Whitney U-test has an impressive asymptotic relative efficiency of 3 relative to the two independent samples *t*-test when the population sampled from is exponential (Conover, 1999). Thus, nonparametric or distribution free tests should be considered when these tests demonstrate both (a) robustness with respect to Type I error and (b) a power advantage relative to the parametric test.

Sawilowsky (1990) reviewed ten competing tests for interaction in the context of factorial ANOVA and ANCOVA. On the basis of Type I error and power properties, three potential competitors to the parametric *F* test remain. These alternative nonparametric tests are: the adjusted RT procedure (Blair & Sawilowsky, 1990; Salter & Fawcett, 1993); the Hettmansperger (1984) procedure; and the Puri and Sen (1969a) procedure. It should be noted that the Hettmansperger (1984) and Puri and Sen (1969a) procedures consider only the total group regression slope. As such, it is assumed that the within group regression slopes are equal for these tests.

Purpose of the Study

The purpose of the study is to compare and contrast the relative Type I error and power properties of the parametric *F* test and the three aforementioned nonparametric procedures in the context of factorial ANCOVA using Monte Carlo techniques. From the results of the Monte Carlo study, a statement will be made with respect to the conditions under which any of the nonparametric tests are useful alternatives to the parametric *F* test. Because good nonparametric tests exist for main effects, the focus of this study is concerned with the test for interaction in the presence and/or absence of main effects.

Methodology

A completely randomized balanced design with fixed effects and one covariate was used. The structural model representing the design was:

$$Y_{ijk} = \mu + \beta(X_{ijk} - \overline{X}) + \alpha_i + \tau_j + (\alpha\tau)_{ij} + \varepsilon_{ijk}, \quad (1)$$

($i = 1,..., I$; $j = 1,..., J$; and $k = 1,..., n$), where $I = 3$, $J = 4$, and $n = 5$, 10, and 20.
The levels of variate (Y_{ijk}) and covariate (X_{ijk}) correlation were ρ = 0, .3, .6, and .9. Note that the regression slope coefficient in (1), β, remained constant across groups.

The treatment effect patterns modeled in (1) were as follows:

1. The main effect τ nonnull, the main effect α null, and the interaction $(\alpha\tau)$ null:
   1(a). $\tau_1 = d$;
   1(b). $\tau_1 = \tau_2 = d$; and $\tau_3 = \tau_4 = -d$.
2. The main effects τ and α nonnull, and the interaction $(\alpha\tau)$ null:
   2(a). $\tau_2 = \alpha_1 = d$; and $\tau_3 = \alpha_2 = -d$; and
   2(b). $\tau_3 = \alpha_1 = d$; and $\tau_1 = \tau_2 = \tau_4 = \alpha_3 = -d$.
3. The $(\alpha\tau)$ interaction nonnull, and the main effects τ and α null:
   3(a). $(\alpha\tau)_{11} = (\alpha\tau)_{33} = d$; and $(\alpha\tau)_{13} = (\alpha\tau)_{31} = -d$;
   3(b). $(\alpha\tau)_{11} = (\alpha\tau)_{14} = (\alpha\tau)_{32} = (\alpha\tau)_{33} = d$; and
   $(\alpha\tau)_{12} = (\alpha\tau)_{13} = (\alpha\tau)_{31} = (\alpha\tau)_{34} = -d$.
4. The main effect τ and the $(\alpha\tau)$ interaction nonnull, and the main effect α null:
   4(a). $(\alpha\tau)_{11} = d$; and $(\alpha\tau)_{14} = -d$;
   4(b). $(\alpha\tau)_{11} = (\alpha\tau)_{12} = (\alpha\tau)_{31} = (\alpha\tau)_{32} = d$; and
   $(\alpha\tau)_{13} = (\alpha\tau)_{14} = (\alpha\tau)_{33} = (\alpha\tau)_{34} = -d$.
5. The main effects τ, α, and $(\alpha\tau)$ interaction are nonnull:
   5(a). $(\alpha\tau)_{21} = (\alpha\tau)_{24} = d$;
   5(b). $(\alpha\tau)_{11} = (\alpha\tau)_{12} = (\alpha\tau)_{32} = (\alpha\tau)_{33} = (\alpha\tau)_{34} = d$; and
   $(\alpha\tau)_{13} = (\alpha\tau)_{31} = (\alpha\tau)_{14} = -d$.

The treatment effect sizes (*d*) ranged from $d = 0.10\sigma$ to $d = 2.00\sigma$, where σ is the standard deviation of the population from which samples were drawn, in increments of 0.10σ. The null case was represented when $d = 0.00$ for all effects.

The parametric *F* statistic was calculated using the OLS sums of squares approach given in Winer, Brown, and Michels (1991) for factorial ANCOVA. The *F* statistic for interaction was then compared to the critical value from the usual *F* tables of percentage points.

The adjusted RT (adjRT) statistic was computed as follows: (a) the residuals were obtained from conducting a two-way ANOVA on the reduced model that included only the grouping variables; (b) the residuals and the covariate were then ranked without respect to group membership; and (c) the usual parametric ANCOVA procedure was conducted on the ranked residuals and ranked covariate to obtain the test statistic for interaction. This statistic was then compared to same critical *F* value as the parametric test.

The Hettmansperger (H) (1984) chi-square statistic was computed as follows: (a) the residuals (*RES*) were obtained from the regression of the variate on the reduced model that included the covariate and the grouping variables; (b) the residuals were then ranked (denoted as *RRES*) without respect to group membership; (c) the standardized ranked residuals (*SRRES*) were obtained according to the following equation: $SRRES = \sqrt{12}\left[\left(\frac{RRES}{N+1} - \frac{1}{2}\right)\right]$;

(d) the *SRRES* were then submitted to a two-way ANOVA; (e) the sums of squares for interaction term obtained from the ANOVA was then compared to the critical value from a chi-square distribution with $(I-1)(J-1)$ degrees of freedom (Hettmansperger, 1984).

The Puri and Sen (PS) (1969a) chi-square statistic was computed as follows: (a) the variate and covariate were ranked irrespective to group membership; (b) the cell means ($\overline{R}_{Yij}, \overline{R}_{Xij}$), column means ($\overline{R}_{Y.j}, \overline{R}_{X.j}$), row means ($\overline{R}_{Yi.}, \overline{R}_{Yi.}$), and overall grand means ($\overline{R}_{Y..}, \overline{R}_{X..}$) were then obtained from the ranks of the variate and covariate scores; (c) the *ij*-th difference score was then obtained as follows:

$$DIFF(R_{Yij}) = (\overline{R}_{Yij} - \overline{R}_{Y..}) - (\overline{R}_{Y.j} - \overline{R}_{Y..}) - (\overline{R}_{Yi.} - \overline{R}_{Y..}) \text{, and}$$

$$DIFF(R_{Xij}) = (\overline{R}_{Xij} - \overline{R}_{X..}) - (\overline{R}_{X.j} - \overline{R}_{X..}) - (\overline{R}_{Xi.} - \overline{R}_{X..});$$

(d) the *ij*-th residual scores were obtained from subtracting the predicted differences from the observed differences as follows: $RES_{ij} = DIFF(R_{Yij)}) - \rho_{YX} DIFF(R_{Xij})$, where ρ_{YX} is the total group rank correlation coefficient between the variate and covariate; (e) the L_n statistic (Puri & Sen, 1969a) was then formulated as: $L_n = \mathbf{V}_{11} \sum_i \sum_j nRES_{ij}^2$, where $\mathbf{V}_{11}$ is the first element on the principal diagonal of the inverted variance-covariance matrix ($\mathbf{V}$); and (f) the computed value of L_n was subsequently compared to the critical value from a chi-square distribution with $(I-1)(J-1)$ degrees of freedom (Puri & Sen, 1969a).

Nine conditional distributions were simulated with zero means ($\mu = 0$), unit variances ($\sigma^2 = 1$), and varying degrees of γ_1, γ_2, γ_3, and γ_4. The distributions approximated in the simulation were: $1=$normal ($\gamma_1 = 0$, $\gamma_2 = 0$, $\gamma_3 = 0$, and $\gamma_4 = 0$), $2=$ uniform ($\gamma_1 = 0$, $\gamma_2 = -6/5$, $\gamma_3 = 0$, and $\gamma_4 = 48/7$); $3=$Cauchy ($\gamma_1 = 0$, $\gamma_2 = 25$, $\gamma_3 = 0$, and $\gamma_4 = 4000$); $4=$double exponential ($\gamma_1 = 0$, $\gamma_2 = 3$, $\gamma_3 = 0$, and $\gamma_4 = 30$); $5=$logistic ($\gamma_1 = 0$, $\gamma_2 = 6/5$, $\gamma_3 = 0$, and $\gamma_4 = 48/7$); $6=$chi-square 8*df* ($\gamma_1 = 1$, $\gamma_2 = 3/2$, $\gamma_3 = 3$, and $\gamma_4 = 15/2$), $7=$chi-square 4*df* ($\gamma_1 = \sqrt{2}$, $\gamma_2 = 4$, $\gamma_3 = 6\sqrt{2}$, and $\gamma_4 = 30$), $8=$chi-square 2*df* ($\gamma_1 = 2$, $\gamma_2 = 6$, $\gamma_3 = 24$, and $\gamma_4 = 120$), and $9=$chi-square 1 *df* ($\gamma_1 = \sqrt{8}$, $\gamma_2 = 12$, $\gamma_3 = 48\sqrt{2}$, and $\gamma_4 = 480$). The preceding values of γ_1 (coefficient of skew), γ_2 (coefficient of kurtosis), γ_3, and γ_4 are the third, fourth, fifth, and sixth standardized cumulants from their associated probability density functions with the exception of the Cauchy distribution. Because the moments of a Cauchy *pdf* are infinite, the above values of γ_1, γ_2, γ_3, and γ_4 associated with this density were selected to yield a symmetric distribution with heavy tail-weight.

The steps employed for data generation follow the model developed by Headrick (2000). The Headrick (2000) procedure is an extension of the Headrick and Sawilowsky (1999, 2000) procedure for simulating multivariate nonnormal distributions. The Headrick (2000) procedure generated the Y_{ijk} and X_{ijk} for the *ij*-th group in (1) from the use of the following equations:

$$Y_{ijk} = c_0 + c_1 Y_{ijk}^{*} + c_2 Y_{ijk}^{*^2} + c_3 Y_{ijk}^{*^3} + c_4 Y_{ijk}^{*^4} + c_5 Y_{ijk}^{*^5} + \delta_{ij} d \text{, and} \tag{2}$$

$$X_{ijk} = c_0 + c_1 X_{ijk}^{*} + c_2 X_{ijk}^{*^2} + c_3 X_{ijk}^{*^3} + c_4 X_{ijk}^{*^4} + c_5 X_{ijk}^{*^5} \text{, where } Y_{ijk}^{*},\ X_{ijk}^{*} \sim \text{iid N(0,1)}. \tag{3}$$

The resulting Y_{ijk} and X_{ijk} were distributed with group means of $\delta_{ij}d$ and zero (respectively), unit variances, the desired values of γ_1, γ_2, γ_3, γ_4, and the desired within group correlation (ρ). In all experimental situations, Y_{ijk} and X_{ijk} followed the same distribution. The value of $\delta_{ij}d$ was the shift parameter added to the *ij*-th group for the treatment effect pattern considered. The coefficients c_0, c_1, c_2, c_3, c_4, and c_5 were determined by simultaneously solving equations 37, 38, 39, 40, 41, and 42 from Headrick (2000) for the desired values of γ_1, γ_2, γ_3, and γ_4. The values of Y^*_{ijk} and X^*_{ijk} in (2) and (3) were generated using the following algorithms:

$$Y^*_{ijk} = Z_{ijk}\rho^* + V_{ijk}\sqrt{1-\rho^{*2}}\text{, and} \tag{4}$$

$$X^*_{ijk} = Z_{ijk}\rho^* + W_{ijk}\sqrt{1-\rho^{*2}}\text{,} \tag{5}$$

where the Z_{ijk}, V_{ijk}, and W_{ijk} ~ iid N(0,1). The resulting Y^*_{ijk} and X^*_{ijk} were normally distributed with zero means, unit variances, and correlated at the intermediate value $\rho^{*^2}_{Y^*_{ijk}X^*_{ijk}}$. The intermediate correlation, which is different from the desired post-correlation ($\rho_{Y_{ijk}X_{ijk}}$) except under conditional normality, was determined by solving equation 26 from Headrick (2000) for the bivariate case for $\rho^*_{Y^*X^*}$. When both variables follow the same distribution, equation 26 from Headrick (2000) can be expressed as follows:

$$\begin{aligned}\rho_{Y_{ijk}X_{ijk}} &= c_0^2 + 9c_4^2 + 2c_0(c_2+3c_4) + c_1^2\rho^{*^2}_{Y^*_{ijk}X^*_{ijk}} + 6c_1c_3\rho^{*^2}_{Y^*_{ijk}X^*_{ijk}} + 9c_3^2\rho^{*^2}_{Y^*_{ijk}X^*_{ijk}} + 30\times \\ & c_1c_5\rho^{*^2}_{Y^*_{ijk}X^*_{ijk}} + 90c_3c_5\rho^{*^2}_{Y^*_{ijk}X^*_{ijk}} + 225c_5^2\rho^{*^2}_{Y^*_{ijk}X^*_{ijk}} + 72c_4^2\rho^{*^4}_{Y^*_{ijk}X^*_{ijk}} + 6c_3^2\rho^{*^6}_{Y^*_{ijk}X^*_{ijk}} + 120c_3c_5\rho^{*^6}_{Y^*_{ijk}X^*_{ijk}} \\ & + 600c_5^2c_5^2\rho^{*^6}_{Y^*_{ijk}X^*_{ijk}} + 24c_4^2\rho^{*^8}_{Y^*_{ijk}X^*_{ijk}} + 120c_5^2\rho^{*^{10}}_{Y^*_{ijk}X^*_{ijk}} + c_2^2(1+2\rho^{*^4}_{Y^*_{ijk}X^*_{ijk}}) + 6c_2(c_4+4c_4\rho^{*^4}_{Y^*_{ijk}X^*_{ijk}})\end{aligned} \tag{6}$$

Values of $c_0,\ldots,c_5$, and $\rho^*_{Y^*_{ijk}X^*_{ijk}}$ were solved for (6) using Mathematica (Version 4.0, 1999). The solution values of $c_0,\ldots,c_5$, the intermediate correlations ($\rho^{*^2}_{Y^*_{ijk}X^*_{ijk}}$), and post-correlations ($\rho_{Y_{ijk}X_{ijk}}$) for the conditional distributions considered are compiled in Table 1.

The computer used to carry out the Monte Carlo was a Pentium III-based personal computer. All programming was done using Lahey Fortran 77 version 3.0 (1994), supplemented with various subroutines from RANGEN (Blair 1986). Using the chi-square and *F* tables of percentage points, the proportions of hypotheses rejected were recorded for the four different procedures. The nominal alpha level selected was .05. Twenty five thousand repetitions were simulated for each of the 9(type of distribution)$\times$4(level of correlation)$\times$21(effect size) $\times$10(treatment effect pattern) experiments.

Results

Adequacy of the Monte Carlo

For each repetition, separate values of ρ_{ij} and $\gamma_{1_{ij}}$, $\gamma_{2_{ij}}$, $\gamma_{3_{ij}}$, and $\gamma_{4_{ij}}$ for the variate and covariate for each of the *IJ* groups were computed. Average values of ρ_{ij} ($\bar{\rho}_{..}$), $\gamma_{1_{ij}}$ ($\bar{\gamma}_{1..}$), $\gamma_{2_{ij}}$ ($\bar{\gamma}_{2..}$), $\gamma_{3_{ij}}$ ($\bar{\gamma}_{3..}$), and $\gamma_{4_{ij}}$ ($\bar{\gamma}_{4..}$) were obtained by averaging the ρ_{ij}, $\gamma_{1_{ij}}$, $\gamma_{2_{ij}}$, $\gamma_{3_{ij}}$, and $\gamma_{4_{ij}}$ across the *IJ* groups. The values of $\bar{\rho}_{..}$, $\bar{\gamma}_{1..}$, $\bar{\gamma}_{2..}$, $\bar{\gamma}_{3..}$, and $\bar{\gamma}_{4..}$ were subsequently averaged across 25,000 (replications) × 21 (effect size) situations in the first treatment effect pattern for each conditional distribution. The average values of $\bar{\gamma}_{1..}$, $\bar{\gamma}_{2..}$, $\bar{\gamma}_{3..}$, and $\bar{\gamma}_{4..}$ were then further averaged across the four levels of correlation. The overall

Table 1. Values of constants ($c_0,\ldots,c_5$) used in equation (3), population correlations ($\rho_{Y_{ijk}X_{ijk}}$), and intermediate correlations ($\rho^{*^2}_{Y^*X^*}$) to simulate and correlate the desired conditional distributions (Dist).

Dist	c_0	c_1	c_2	c_3	c_4	c_5	$\rho_{Y_{ijk}X_{ijk}}$	$\rho^{*^2}_{Y^*X^*}$
1	0.000000	1.000000	0.000000	0.000000	0.000000	0.000000	.00	.000000
							.30	.300000
							.60	.700000
							.90	.900000
2	0.000000	1.347438	0.000000	-0.140177	0.000000	0.001808	.00	.000000
							.30	.326197
							.60	.634118
							.90	.913613
3	0.000000	0.306093	0.000000	0.184686	0.000000	0.001132	.00	.000000
							.30	.374236
							.60	.683980
							.90	.929263
4	0.000000	0.727709	0.000000	0.096303	0.000000	-0.002232	.00	.000000
							.30	.309371
							.60	.612882
							.90	.905531
5	0.000000	0.879467	0.000000	0.040845	0.000000	-0.000405	.00	.000000
							.30	.302233
							.60	.603260
							.90	.901368
6	-0.163968	0.950794	0.165391	0.007345	-0.000474	0.000014	.00	.000000
							.30	.311431
							.60	.612677
							.90	.904625
7	-0.227508	0.900716	0.231610	0.015466	-0.001367	0.000055	.00	.000000
							.30	.322263
							.60	.624030
							.90	.908552
8	-0.307740	0.800560	0.318764	0.033500	-0.003675	0.000159	.00	.000000
							.30	.341958
							.60	.643339
							.90	.914879
9	-0.397725	0.621071	0.416907	0.068431	-0.006394	0.000044	.00	.000000
							.30	.376853
							.60	.673908
							.90	.924127

averages of $\overline{\gamma}_1$, $\overline{\gamma}_2$, $\overline{\gamma}_3$, $\overline{\gamma}_4$, and $\overline{\rho}$ are listed in Table 2 and Table 3, respectively. Inspection of Tables 2 and 3 indicate that the Headrick (2000) procedure produced excellent agreement between $\overline{\gamma}_1$, $\overline{\gamma}_2$, $\overline{\gamma}_3$, $\overline{\gamma}_4$, and $\overline{\rho}$ and the population parameters considered.

The Type I error and power analyses are compiled in Tables 4 through 13. The column entries from left to right denote (a) the test statistic, (b) the standardized treatment effect size "*d*", and (c) the proportion of rejections for the four different tests of interaction under the various levels of variate and covariate correlation and the other parameters considered.

Type I Error

Normal Distribution: The Type I error rates for the competing procedures are compiled in Tables 4, 6, and 8, for $n = 5$, 10, 20, and treatment pattern 2(b). This particular effect pattern is reported because the commonly used rank transform test statistic (Conover & Iman, 1981) under these circumstances is not

Table 2. Average values of $\gamma_1(\bar{\gamma}_1)$, $\gamma_2(\bar{\gamma}_2)$, $\gamma_3(\bar{\gamma}_3)$, and $\gamma_4(\bar{\gamma}_4)$ simulated by the Headrick (2000) procedure. The average values ($\bar{\gamma}_1$, $\bar{\gamma}_2$, $\bar{\gamma}_3$, $\bar{\gamma}_4$) listed below were based on a sample size is $n=20$.

Distribution	Population parameter (γ_1, γ_2, γ_3, γ_4)			
1	$\gamma_1=0$	$\gamma_2=0$	$\gamma_3=0$	$\gamma_4=0$
Variate (Y)	$\bar{\gamma}_1=0.000124$	$\bar{\gamma}_2=-0.000284$	$\bar{\gamma}_3=0.001073$	$\bar{\gamma}_4=-0.001339$
Covariate (X)	$\bar{\gamma}_1=-0.000084$	$\bar{\gamma}_2=0.000452$	$\bar{\gamma}_3=0.000795$	$\bar{\gamma}_4=0.002845$
2	$\gamma_1=0$	$\gamma_2=-6/5$	$\gamma_3=0$	$\gamma_4=48/7$
Variate (Y)	$\bar{\gamma}_1=0.000005$	$\bar{\gamma}_2=-1.200004$	$\bar{\gamma}_3=0.0000238$	$\bar{\gamma}_4=6.857894$
Covariate (X)	$\bar{\gamma}_1=0.000039$	$\bar{\gamma}_2=-1.200163$	$\bar{\gamma}_3=0.0001685$	$\bar{\gamma}_4=6.853492$
3	$\gamma_1=0$	$\gamma_2=25$	$\gamma_3=0$	$\gamma_4=4000$
Variate (Y)	$\bar{\gamma}_1=-0.001318$	$\bar{\gamma}_2=24.975520$	$\bar{\gamma}_3=-.3386690$	$\bar{\gamma}_4=3958.22114$
Covariate (X)	$\bar{\gamma}_1=0.000290$	$\bar{\gamma}_2=24.941770$	$\bar{\gamma}_3=-0.799517$	$\bar{\gamma}_4=3988.30400$
4	$\gamma_1=0$	$\gamma_2=3$	$\gamma_3=0$	$\gamma_4=30$
Variate (Y)	$\bar{\gamma}_1=0.000342$	$\bar{\gamma}_2=2.999848$	$\bar{\gamma}_3=0.014447$	$\bar{\gamma}_4=30.010830$
Covariate (X)	$\bar{\gamma}_1=0.000032$	$\bar{\gamma}_2=3.000327$	$\bar{\gamma}_3=0.004328$	$\bar{\gamma}_4=30.006732$
5	$\gamma_1=0$	$\gamma_2=6/5$	$\gamma_3=0$	$\gamma_4=48/7$
Variate (Y)	$\bar{\gamma}_1=0.000224$	$\bar{\gamma}_2=1.199900$	$\bar{\gamma}_3=.004258$	$\bar{\gamma}_4=6.846827$
Covariate (X)	$\bar{\gamma}_1=0.000034$	$\bar{\gamma}_2=1.200087$	$\bar{\gamma}_3=.001478$	$\bar{\gamma}_4=6.858595$
6	$\gamma_1=1$	$\gamma_2=3/2$	$\gamma_3=3$	$\gamma_4=15/2$
Variate (Y)	$\bar{\gamma}_1=1.000071$	$\bar{\gamma}_2=1.500197$	$\bar{\gamma}_3=3.001597$	$\bar{\gamma}_4=7.496629$
Covariate (X)	$\bar{\gamma}_1=0.999992$	$\bar{\gamma}_2=1.500053$	$\bar{\gamma}_3=3.005218$	$\bar{\gamma}_4=7.538564$
7	$\gamma_1=\sqrt{2}$	$\gamma_2=3$	$\gamma_3=6\sqrt{2}$	$\gamma_4=30$
Variate (Y)	$\bar{\gamma}_1=1.414330$	$\bar{\gamma}_2=3.000764$	$\bar{\gamma}_3=8.489000$	$\bar{\gamma}_4=29.978800$
Covariate (X)	$\bar{\gamma}_1=1.413904$	$\bar{\gamma}_2=3.001067$	$\bar{\gamma}_3=8.484897$	$\bar{\gamma}_4=30.004765$
8	$\gamma_1=2$	$\gamma_2=6$	$\gamma_3=24$	$\gamma_4=120$
Variate (Y)	$\bar{\gamma}_1=2.000254$	$\bar{\gamma}_2=6.002129$	$\bar{\gamma}_3=24.008980$	$\bar{\gamma}_4=119.868700$
Covariate (X)	$\bar{\gamma}_1=1.999989$	$\bar{\gamma}_2=6.000573$	$\bar{\gamma}_3=24.010045$	$\bar{\gamma}_4=120.158647$
9	$\gamma_1=\sqrt{8}$	$\gamma_2=12$	$\gamma_3=48\sqrt{2}$	$\gamma_4=480$
Variate (Y)	$\bar{\gamma}_1=2.828878$	$\bar{\gamma}_2=12.003800$	$\bar{\gamma}_3=67.884840$	$\bar{\gamma}_4=479.035600$
Covariate (X)	$\bar{\gamma}_1=2.827901$	$\bar{\gamma}_2=12.000050$	$\bar{\gamma}_3=67.885672$	$\bar{\gamma}_4=480.001874$

asymptotically chi-squared (Thompson, 1991, 1993) and is liberal for even small samples (Headrick, 1997; Headrick & Sawilowsky, 2000).

As expected, the parametric F test maintained Type I error rates close to nominal alpha and were within the closed interval of $\alpha \pm 1.96\sqrt{\alpha(1-\alpha)/25000}$. This occurred across all treatment conditions, sample sizes, and levels of variate/covariate correlation.

The adjRT also generated acceptable Type I error rates. Inspection of Tables 4, 6, and 8 indicates that the Type I error rates were similar to the parametric F test. With respect to the H test, inspection of Tables 6 and 8 indicates that this test maintained appropriate Type I error rates for sample sizes of $n=10$ and $n=20$. However, for $n=5$, inspection of Table 4 indicates that the H test generated liberal Type I error rates. For example, with an effect size of $d=0.80$, the Type I error rates were approximately .060 across all levels of variate/covariate correlation.

Table 3. Average values of variate and covariate correlation ($\bar{\rho}$) simulated by the Headrick (2000) procedure. The value ρ denotes the population correlation. The average values ($\bar{\rho}$) listed below were based on a sample size is $n=20$.

		Distribution								
n	ρ	1	2	3	4	5	6	7	8	9
20	.00	.000	.000	-.001	.000	.000	.000	.001	.000	-.000
	.30	.300	.299	.300	.301	.300	.299	.300	.300	.301
	.60	.600	.601	.602	.599	.600	.598	.600	.599	.600
	.90	.900	.899	.901	.900	.900	.901	.899	.900	.900

The PS test became conservative when either one or both main effects were present. *Ceteris paribus*, the stronger the nonnull main effect(s) the more conservative the Type I error rates became. These conservative Type I error rates occurred across all levels of variate and covariate correlation. For example, with an effect size of $d=0.80$, inspection of Table 4 indicates that the Type I error rates were .001, .000, and .000 across the three levels of variate/covariate correlation. The PS procedure maintained Type I error rates close to nominal alpha only when *both* main effects were null.

Nonnormal Distributions: Type I error rates are compiled in Tables 10 and 12 for some of the nonnormal distributions considered. The approximate distributions reported in these tables are the chi-square 1*df* and Cauchy. These distributions are reported because previous empirical investigations demonstrated that Type I error inflations associated with the rank transform test statistic (Conover & Iman, 1981) were most severe under extreme departures from normality (Headrick, 1997; Headrick & Sawilowsky, 2000).

The parametric F test was slightly conservative under the nonnormal conditional distributions reported. For example, with an effect size of $d=1.30$, variate/covariate correlation of $r=.30$, an inspection of Table 12 indicates that the Type I error rate was .040 when the conditional distribution was approximate Cauchy.

The adjRT generated inflated Type I error rates when the conditional distribution considered was skewed (e.g., chi-square 1*df* or 2*df*). For example, with an effect size of $d=0.80$, a variate/covariate correlation of $r=.90$, inspection of Table 10 indicates that the Type 1 error rate for the adjRT was .076. In general, increases in skew i.e., chi-square 4*df*, chi-square 2*df*, chi-square 1*df* were associated with increases in Type I error inflation for the adjRT.

The H test maintained appropriate Type I error rates for all nonnormal conditional distributions considered when sample sizes were $n=10$ and $n=20$. When samples were $n=5$, the H test generated liberal Type I error rates. The inflated Type I error rates were similar to those error rates generated under conditional normality.

As with the standard normal case, the PS test generated ultra-conservative Type I error rates when main effects were present. For example, with an effect size $d=0.80$ and a variate/covariate correlation of $r=.60$, inspection of Table 12 indicates that the Type I error rate was .000. This occurred for *all* nonnormal distributions considered in this study.

Power Analysis

Normal Distribution: Power analyses for the competing procedures are compiled in Tables 5, 7, and 9, for $n=5$, 10, 20, and treatment pattern 5(a). This effect pattern is reported because under these conditions the usual rank transform statistic has been demonstrated to display severe power losses (Headrick, 1997; Headrick & Sawilowsky, 2000).

As expected, the F test displayed a power advantage over the three nonparametric competitors when the conditional distribution was standard normal. Specifically, the F test was substantially more powerful than the PS test when both main effects became increasingly nonnull. Although the F test was more powerful than the H test when sample sizes were $n=10$ and $n=20$, the H test held a slight power advantage over the adjRT. When sample sizes were $n=5$, inspection of Table 5 indicates that the H test

Table 4. Type I error results for the test of interaction. The sampling distribution was standard normal. The sample size was $n = 5$. Both main effects were nonnull. The Type I error rates were based on 25,000 repetitions and a nominal alpha level of $\alpha = .05$.

		Level of Correlation		
Test	Effect Size (d)	0.3	0.6	0.9
F	0.30	.051	.052	.050
adjRT		.052	.053	.051
H		.057	.060	.060
PS		.023	.020	.004
F	0.80	.050	.051	.050
adjRT		.052	.053	.049
H		.059	.056	.058
PS		.001	.000	.000
F	1.30	.052	.047	.052
adjRT		.052	.050	.051
H		.059	.060	.061
PS		.000	.000	.000

Table 5. Power analysis for the test of interaction when sampling was from a standard normal distribution. The sample size was $n = 5$. Both main effects were nonnull. The rejection rates were based on 25,000 repetitions and a nominal alpha level of $\alpha = .05$.

		Level of Correlation		
Test	Effect Size (d)	0.3	0.6	0.9
F	0.30	.062	.067	.110
adjRT		.062	.066	.100
H		.069	.077	.121
PS		.055	.056	.081
F	0.80	.145	.202	.622
adjRT		.143	.187	.531
H		.159	.217	.632
PS		.106	.132	.315
F	1.30	.359	.507	.983
adjRT		.349	.473	.954
H		.372	.517	.983
PS		.211	.272	.272

was rejecting at a higher rate than *F* test. For example, with an effect size of $d = 0.80$, a variate/covariate correlation of $r = .30$, inspection of Table 5 indicates that the H test had a rejection rate of .16 while the *F* test was rejecting at a rate of .145. This higher rejection rate is attributed to the liberal nature of the Type I error rates that were associated with the H test when $n = 5$.

Nonnormal Distributions: In general, when departures from normality were small (e.g., approximate logistic) to moderate (e.g., approximate chi-square 8*df*) the *F* test rejected at rates slightly less than the Hettmansperger and adjRT procedures. The power advantages in favor of either the H or adjRT tests were contingent on the conditional distribution considered and the other parameters being simulated. It should be noted that the power advantages in favor either the H test or adjRT test were marginal. On the other hand, when the conditional distribution was approximate uniform the parametric *F* test held a slight advantage over the nonparametric procedures.

When the conditional distributions were extremely skewed and/or heavy tailed, both the adjRT and H tests held large power advantages over the *F* test. Further, when the adjRT test generated reasonable Type I error rates, the adjRT displayed some power advantages over the other competing nonparametric procedures. For example, inspection of Table 13 indicates that when the conditional distribution was approximate Cauchy, an effect size of $d = 0.80$, and a variate/covariate correlation of $r = .60$, the adjRT

Table 6. Type I error results for the test of interaction. The sampling distribution was standard normal. The sample size was $n = 10$. Both main effects were nonnull. The Type I error rates were based on 25,000 repetitions and a nominal alpha level of $\alpha = .05$.

		Level of Correlation		
Test	Effect Size (*d*)	0.3	0.6	0.9
F	0.30	.053	.052	.049
adjRT		.054	.051	.049
H		.052	.055	.053
PS		.027	.019	.003
F	0.80	.050	.050	.050
adjRT		.051	.049	.051
H		.053	.052	.053
PS		.006	.001	.000
F	1.30	.050	.048	.050
adjRT		.051	.048	.051
H		.054	.051	.054
PS		.000	.000	.000

Table 7. Power analysis for the test of interaction when sampling was from a standard normal distribution. The sample size was $n = 10$. Both main effects were nonnull. The rejection rates were based on 25,000 repetitions and a nominal alpha level of $\alpha = .05$.

		Level of Correlation		
Test	Effect Size (*d*)	0.3	0.6	0.9
F	0.30	.078	.088	.208
adjRT		.077	.087	.179
H		.076	.089	.201
PS		.069	.077	.155
F	0.80	.294	.418	.951
adjRT		.284	.386	.911
H		.288	.402	.943
PS		.232	.306	.779
F	1.30	.715	.879	1.000
adjRT		.693	.848	.999
H		.697	.863	1.000
PS		.531	.683	.987

was rejecting at a rate of .942 whereas the H test was rejecting at a rate of .844. Power comparisons between these two tests were not considered where the adjRT generated liberal Type error rates (e.g., approximate chi-square 2*df* or chi-square 1*df*). When the conditional distributions were approximately chi-square 2*df* or chi-square 1*df*, the H test was a much more powerful than the parametric *F*. For example, when sampling was from an approximate chi-square distribution 1*df*, $d = 0.80$, a variate/covariate correlation of $r = .30$, inspection of Table 11 indicates that the H test was rejecting at a rate of .731 while the F test was rejecting at a rate of only .326.

The PS procedure held a power advantage over the H and adjRT tests only when *both* main effects were either weak or null. Otherwise, the PS test statistic had the problem of power loss when juxtaposed to either the H or the adjRT tests as the magnitude of the main effect(s) increased. For example, when sampling was from an approximate chi-square distribution 1*df*, $d = 0.30$, a variate/covariate correlation of $r = .30$, inspection of Table 11 indicates that the PS test was rejecting at a rate of .182 while the H test was rejecting at a rate of .148. However, when the effect size increased from $d = .30$ to $d = 0.80$, the H test was rejecting at a rate of .731 while the PS was rejecting at a rate of only .524. This pattern of power loss associated with the PS test was consistent across all nonnormal distributions considered in this study.

Table 8. Type I error results for the test of interaction. The sampling distribution was standard normal. The sample size was $n = 20$. Both main effects were nonnull. The Type I error rates were based on 25,000 repetitions and a nominal alpha level of $\alpha = .05$.

		Level of Correlation		
Test	Effect Size (*d*)	0.3	0.6	0.9
F	0.30	.050	.050	.049
adjRT		.050	.050	.050
H		.051	.052	.050
PS		.028	.019	.003
F	0.80	.051	.049	.052
adjRT		.052	.052	.051
H		.052	.052	.052
PS		.001	.000	.000
F	1.30	.050	.050	.050
adjRT		.050	.049	.048
H		.052	.051	.052
PS		.000	.000	.000

Table 9. Power analysis for the test of interaction when sampling was from a standard normal distribution. The sample size was $n = 20$. Both main effects were nonnull. The rejection rates were based on 25,000 repetitions and a nominal alpha level of $\alpha = .05$.

		Level of Correlation		
Test	Effect Size (*d*)	0.3	0.6	0.9
F	0.30	.109	.133	.410
adjRT		.105	.127	.360
H		.105	.131	.393
PS		.099	.119	.328
F	0.80	.596	.775	1.000
adjRT		.569	.734	1.000
H		.573	.754	1.000
PS		.505	.661	.994
F	1.30	.976	.998	1.000
adjRT		.968	.996	1.000
H		.969	.997	1.000
PS		.920	.978	1.000

Discussion

The PS test is computationally arduous. Further, the results of this study indicate that this test had the problems of ultra-conservative Type I error rates and power loss when main effects were nonnull. Toothaker and Newman (1994) found similar results with respect to the PS test in the context of factorial ANOVA. Thus, it is recommended that this procedure not be considered as a viable alternative to the parametric *F* test in factorial ANCOVA.

It is possible to base the PS statistic on normal or expected normal scores instead of the ranks (Puri & Sen, 1969a). And, this *might* correct the problem of ultra-conservative Type I error rates. However, additional nonlinear transformations present the problem with respect to the correct interpretation of the statistical results in terms of the original metric.

The adjRT is arguably the simplest of the three nonparametric procedures to compute. However, because the adjRT has the problem of liberal Type I error rates when the distributions possess moderate to extreme skewness, it is also recommended that the adjRT procedure not be used in place of the parametric *F* test.

Table 10. Type I error results for the test of interaction. The sampling distribution was an approximate chi-square distribution with 1degree of freedom. The sample size was $n = 10$. Both main effects were nonnull. The Type I error rates were based on 25,000 repetitions and a nominal alpha level of $\alpha = .05$.

		Level of Correlation		
Test	Effect Size (*d*)	0.3	0.6	0.9
F	0.30	.046	.043	.045
adjRT		.069	.068	.075
H		.053	.048	.048
PS		.010	.006	.001
F	0.80	.044	.047	.047
adjRT		.067	.072	.076
H		.051	.050	.049
PS		.004	.001	.000
F	1.30	.045	.046	.046
adjRT		.070	.070	.073
H		.052	.049	.049
PS		.000	.000	.000

Table 11. Power analysis for the test of interaction when sampling was from was an approximate chi-square distribution with 1degree of freedom. The sample size was $n = 10$. Both main effects were nonnull. The rejection rates were based on 25,000 repetitions and a nominal alpha level of $\alpha = .05$.

		Level of Correlation		
Test	Effect Size (*d*)	0.3	0.6	0.9
F	0.30	.075	.086	.221
adjRT		.197	.262	.679
H		.148	.182	.522
PS		.182	.243	.597
F	0.80	.326	.462	.947
adjRT		.815	.926	.999
H		.731	.838	.999
PS		.524	.672	.973
F	1.30	.739	.881	.999
adjRT		.991	.999	1.000
H		.981	.995	1.000
PS		.762	.885	.998

The H chi-square test maintained appropriate Type I error rates for *all* conditional distributions considered in this study when sample sizes were at least as large as $n = 10$. Thus, the H test could be considered as an alternative to the parametric *F* test for interaction provided the within group sample sizes are relatively equal and *at least as large* as $n = 10$. This recommendation is made in view of the large power advantage that the H test had over the *F* test when the conditional distributions were contaminated with outliers and/or possessed extreme skewness.

Table 12. Type I error results for the test of interaction. The sampling distribution was an approximate Cauchy distribution. The sample size was $n = 10$. Both main effects were nonnull. The Type I error rates were based on 25,000 repetitions and a nominal alpha level of $\alpha = .05$.

		Level of Correlation		
Test	Effect Size (*d*)	0.3	0.6	0.9
F	0.30	.043	.045	.044
adjRT		.054	.053	.058
H		.046	.050	.050
PS		.008	.005	.001
F	0.80	.044	.044	.046
adjRT		.053	.055	.056
H		.045	.048	.048
PS		.000	.000	.000
F	1.30	.040	.044	.045
adjRT		.053	.052	.056
H		.045	.047	.048
PS		.000	.000	.000

Table 13. Power analysis for the test of interaction when sampling was from was an approximate Cauchy distribution. The sample size was $n = 10$. Both main effects were nonnull. The rejection rates were based on 25,000 repetitions and a nominal alpha level of $\alpha = .05$.

		Level of Correlation		
Test	Effect Size (*d*)	0.3	0.6	0.9
F	0.30	.075	.096	.244
adjRT		.163	.235	.699
H		.130	.173	.495
PS		.155	.220	.632
F	0.80	.346	.489	.946
adjRT		.801	.942	1.00
H		.712	.844	.999
PS		.563	.754	.993
F	1.30	.750	.884	.999
adjRT		.993	.999	1.00
H		.981	.996	1.00
PS		.804	.934	.999

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Commonality Analysis: Understanding Variance Contributions to Overall Canonical Correlation Effects of Attitude Toward Mathematics on Geometry Achievement

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Canonical correlation analysis is the most general linear model subsuming all other univariate and multivariate cases (Kerlinger & Pedhazur, 1973; Thompson, 1985, 1991). Because "reality" is a complex place, a multivariate analysis such as canonical correlation analysis is demanded to match the research design. It is the purpose of this paper to increase the awareness and use of canonical correlation analysis and, specifically to demonstrate the value of the related procedure of commonality analysis. Commonality analysis provides the researcher with information regarding the variance explained by each of the measured variables and the common contribution from one or more of the other variables in a canonical analysis (Beaton, 1973; Frederick, 1999). This paper identifies confidence as contributing the most unique variance to the model, being more important than either intrinsic value or worry to geometry content knowledge and spatial visualization.

In developing the concept of commonality analysis (CA) one must be familiar with canonical correlation analysis (CCA), a multivariate technique. Most educational research settings demand an analysis that accounts for reality so a multivariate analysis should be used to match the research design as closely as possible. Canonical correlation analysis (CCA) is the most general case of the general liner model (GLM) (Baggaley, 1981). All univariate and multivariate cases can be treated as special cases of CCA (Thompson, 1984, 1991). As Henson (2000) noted, "CCA is superior to ANOVA and MANOVA when the independent variables are intervally scaled, thus eliminating the need to discard variance" otherwise one should refrain from using canonical correlation for these purposes.

There are several rational reasons for selecting CCA. Regarding OVA methods, the first is that CCA honors the relationship among variables because CCA does not require the variables to be converted from their original scale into arbitrary predictor categories (Frederick, 1999). Second, the method honors the reality to which the researcher is often trying to generalize (Henson, 2000; Tatsuoka, 1971; Thompson, 1984,1991). Third, reality has multiple outcomes with multiple causes; thus, it follows that most causes have multiple effects necessitating a multivariate approach (Thompson, 1991). Therefore, any analytic model that does not account for reality in which research is conducted distorts interpretations and potentially provides unreliable results (Tatsuoka, 1971). Historicalyr, research studies rarely used CCA. Prohibitive calculations, difficulty in trying to interpret canonical results and general unfamiliarity with the method contributed to CCA's absence from the literature (Baggaley, 1981; DeVito, 1976; Fan, 1996; Thompson, 1984).

Using CCA in real-life research situations increases the reliability of the results by limiting the inflation of Type I "experimentwise" error rates by reducing the number of analyses in a given study (Shavelson, 1988; Thompson, 1991). As Thompson (1991) stated CCA's limitation of "experimentwise" error, reduces the probability of making a Type I error anywhere within the investigation. Commonly, Type I error refers to "testwise" error rates, the probability of making an error in regards to a specified hypothesis test.

Thompson (1984) stated that some research almost demands CCA in which " . . . it is the simplest model that can do justice to the difficult problem of scientific generalization" (p. 8). Furthermore, the use of CCA leads to the use of commonality analysis (Thompson, 1984). Although the voluminous output from CCA can be difficult to interpret (Tatsuoka, 1971; Thompson, 1984, 1990), however, once complete and noteworthy results emerge, one is obliged to consider the use of commonality analysis.

Commonality Analysis

Commonality analysis, also known as elements analysis and components analysis was developed for multiple regression analysis in the late 1960's (Newton & Spurell, 1967; Thompson, Miller, & James, 1985). Commonality analysis provides the researcher with information regarding the variance explained by each of the measured variables and the common contribution from one or more of the other variables (Beaton, 1973; Frederick, 1999). Partitioning of the variables takes two distinct forms. The first is in the form of explanatory ability that is in common with other variable(s). The second explanatory power can be attributed to unique contributions of a variable. This information should not be confused with interaction effects of regression. Interaction effects cannot be considered as indicating a unique contribution to the criterion set. Each variable in the predictor set simply adds predictive ability or increased variance to the first one (variable) entered. Commonality analysis, however, determines the variance explained that two or more predictor variables share that is useful in predicting relationships with the criterion variable set. Essentially, Beaton (1973) stated that CA partitions the common and unique variance of the several possible predictor variables on the set of criterion variables.

Commonalities can be either positive or negative. Beaton (1973) explained that negative commonalities are rare in educational research but more common in physical science research. While both positive and negative commonalities are useful, negative commonalities indicate that one variable confounds the variance explained by another. When referring to the power of CA, power is synonymous with variance explained. Negative commonalities may actually indicate improved power when both variables are used to make predictions (Beaton, 1973). The following example illustrates the relationship: An Olympic track athlete must be fast and strong, therefore, a strong-fast athlete would be correlated with success at running track. However, one would believe the two variables (fast and strong) would be moderately negatively correlated, that is as muscle strength and mass increase, speed would decrease. The negative commonality between speed and strength would indicate a confounded variable. In this case, by knowing both the speed and strength one would expect to make better predictions of successful track running. Imagine just knowing the speed or strength of the athlete. A fast athlete may perform well in a short sprint but be severely impaired in a distance event. Conversely, a strong athlete may excel in endurance and persevere for distance, but lack the speed to win. The negative commonality in this case indicates that the power of both variables is greater when the other variable is also used.

Conducting a Commonality Analysis

The complexity of conducting a CA ranges from the unsophisticated to the sublime. Frederick (1999) suggested the use of no more than four (predictor) measured variables because as the number of predictors increases so does the difficulty of interpretation. Frederick continued, explaining that the commonality calculations increase in difficulty exponentially as the number of predictors increases. Pedhazur (1982) and Frederick (1999) recommend that to avoid some of the complexities one should group similar variables or do some preliminary analyses to distinguish the most powerful predictors before conducting the CA such as a canonical correlation analysis.

The full model CCA is run with the following SPSS syntax:

```
MANOVA
spacerel gcksum with int.val worry confid
/print=signif (multiv eigen dimenr)
/discrim=(stan estim cor alpha(.999))/design.
```

The criterion variables are space relations (spacerel) and geometry content knowledge (gcksum). The predictor variables are confidence solving mathematics problems (confid), worry (worry), and finally mathematics intrinsic value (int.val). Possible relationships among variables are illustrated by Figure 1.

The Venn diagram illustrating commonality analysis in Figure 1 serves as a model for the comparison of data examined in the present paper. The data was collected in a southeastern state and represents 287 sixth grade students' scores on three measures, the *Space Relations* portion of the *Differential Aptitude Test* (Bennett, Seashore, & Wesman, 1973), the *Geometry Content Knowledge* test (Carroll, 1998), and the *Mathematics Attitude Scale* (Gierl & Bisanz, 1997).

The first step in running a CA begins with the findings of the CCA (the syntax provided earlier; also see the Appendix for the complete SPSS syntax). The next step involves running a descriptive analysis for the purposes of obtaining the standard deviation and means of each variable in order to calculate *z*-scores. The *z*-scores are computed for the observed variables by the following SPSS syntax:

```
COMPUTE zspace = (spacerel- mean)/standard deviation.
COMPUTE zgck = (gcksum- mean)/standard deviation.
```

To create the synthetic canonical variate scores, multiply the z-scores by the standardized canonical function coefficients (found in the original CCA), and then sum the scores for the function. The following SPSS syntax will yield the two sets of criterion variable composite scores (called crit1 and crit2) for both canonical functions.

```
COMPUTE crit1=(standardized canonical function coefficient I*zspace)
        +(standardized canonical function coefficient I*zgck).
COMPUTE crit2=(standardized canonical function coefficient II*zspace)
        +(standardized canonical function coefficient II*zgck).
```

Next, the CA requires running several multiple regression analyses for each criterion composite i.e., crit1 and crit2 using all possible combinations of predictor variables. Refer to Table 1 for the combinations for 2 or 3 predictor variables.

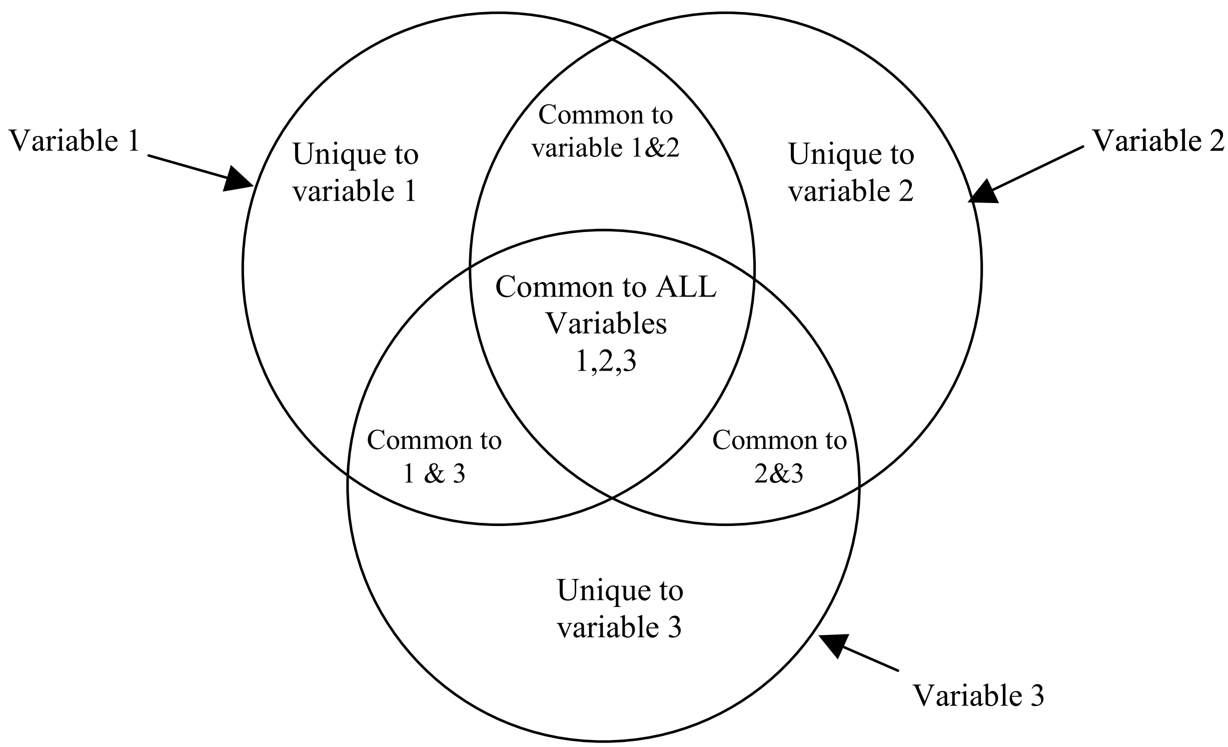

Figure 1. *Illustrating Commonality Analysis.*

Table 1. *Methods of Computing Unique and Common Variance.*

Two Predictor Variables
$U(1)=R^2_{12}-R^2_2$, $U(2)=R^2_{12}-R^2_1$, $C(12)=R^2_2+R^2_1-R^2_{12}$
Three Predictor Variables
$U(1)=R^2_{123}-R^2_{23}$, $U(2)=R^2_{123}-R^2_{13}$, $U(3)=R^2_{123}-R^2_{12}$, $C(12)= R^2_{13}-R^2_3+R^2_{23}-R^2_{123}$, $C(13)= R^2_{12}-R^2_2+R^2_{23}-R^2_{123}$, $C(23)= R^2_{12}-R^2_1+R^2_{13}-R^2_{123}$, $C(123)= R^2_{12}-R^2_2+R^2_3-R^2_{12}-R^2_{13}-R^2_{23}+R^2_{123}$

Note: U= unique variance, C= common variance, C13 = Common to variables 1 & 3
R^2=squared multiple correlation from the respective regression analysis.

Table 2. *Commonality Table.*

Variance	Function I				Function II			
Partition	Intrinsic	Worry	Confidence	Composite	Intrinsic	Worry	Confidence	Composite
U Intrinsic	0.001	0.001			0.019			0.019
U Worry	0.009		0.009		0	0		
U Confidence	0.188			0.188	.003		.003	
C IW	0.002	0.002			0.002			0.002
C IC	0.049	0.049	0.049	0.049	-0.003			-0.003
C WC	0.004		0.004	0.004	0			
C IWC	-0.011	-0.011	-0.011	-0.011	0	0	0	0
R^2 with Crit	0.041	0.004	0.230	0.242	0.018	0.002	0	0.021

Table 3. *Comparisons of Multivariate CCA and Univariate Multiple Regression with All Predictors.*

	Function	
Statistic	I	II
Multiple Regression (R^2)	.242	.021
Canonical Correlation (Rc^2)	.242	.021

Finally, add or subtract relevant effects to calculate the unique and common variance components for each predictor variable on each composite. Do this either by hand or by spreadsheet. The number of components in an analysis will equal (2^k-1), where k= number of predictor variables in the set. So, four predictors produce, 15 components, four-first order (unique), six-second order (common to two variables), four-third order (common to three variables), and one- fourth order (common to all).

The analysis of the present data consisted of two criterion variables, space relations and geometry content knowledge, and three predictor variables from the subscales of the *Mathematics Attitude Scales,* confidence, worry, and intrinsic value. One would expect, that through the application of (2^k-1), to have seven composites, three-first order (unique), three-second order (common to two) and one-third order (common to all). Results are displayed in Table 2.

Recall that both a full CCA and multiple linear regression with all predictors were conducted. The results displayed in Table 3 confirm that both procedures yielded the same results. Note that the R^2 and Rc^2 for Functions I and II are the same for both the multiple regression and CCA. The R^2 from the multiple linear regression reflect the additive effects of all the predictor combinations. These numbers will be confirmed again when summing all of the separate composites for each function (Table 2).

Analyzing Results

One must return to the Venn diagram (Figure 1) and then reconstruct it using the actual data from Table 2. This graphic helps one to visualize the relationships of the partitioned variance. If one only requires the variance explained from the entire CCA then there is no need to conduct a CA. However, the

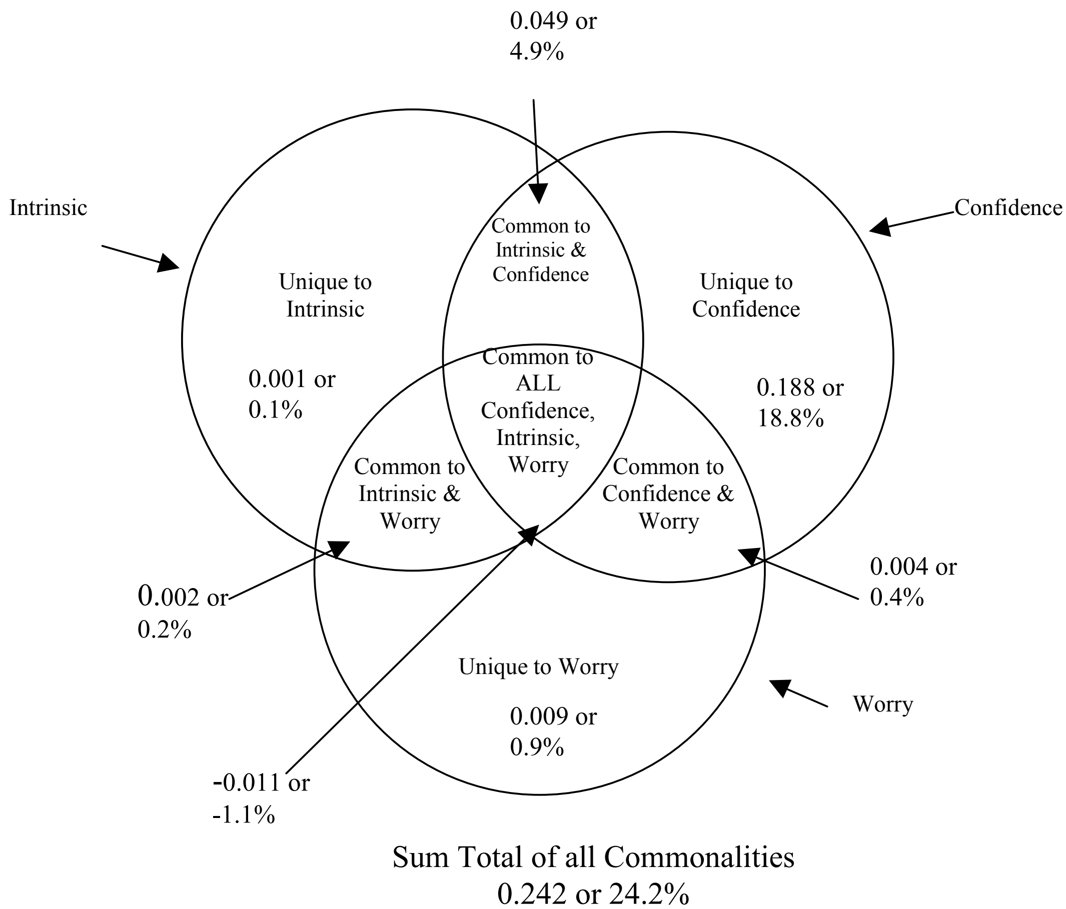

Figure 2. *Venn Diagram Showing Commonalities for Function I.*

power to partition the variance and observe which variable contributes what variance is invaluable when determining parsimony. In analyzing the data from Function I, one notices that confidence explains 18.8% of the variance alone while intrinsic value and confidence contribute 4.9 % in common. The three predictors when taken together explained 24.2% of the first function. Worry and intrinsic value explain very little of the variance from Function I, either uniquely (0.9% or 0.1%) or in common (0.1% to 0.4%) with other measured predictor variables.

Frederick (1999) stated that negative commonalities should be interpreted as zero. While Beaton (1973) believed that negative commonalities were actually confounding, increasing the predictive ability. Caution needs to be taken when interpreting the negative commonality in the common to all variables (Figure 2). As stated before in the analogy to the athlete, a negative commonality on one variable may improve the overall prediction power. However, in this case it is more appropriate to interpret the negative commonality as zero. Think of the situation this way, the variance explained by all three variables inversely predicts the variance explained when all the variables are taken separately. This scenario makes little sense and implies that the variables as a whole indicate an inverse relationship to the criterion variables where they imply a direct relationship when considered individually.

In Function I, summing the variance explained from each of the unique variables and each of the common contributions yields 0.242. The 0.242 is the variance explained in the multiple regression (R^2) and the canonical correlation Rc^2. Because CA yields the partitioned values, one would expect that the sum of the values would equal the total variance explained by either the univariate or the multivariate approach. This also illustrates that CCA subsumes the univariate case.

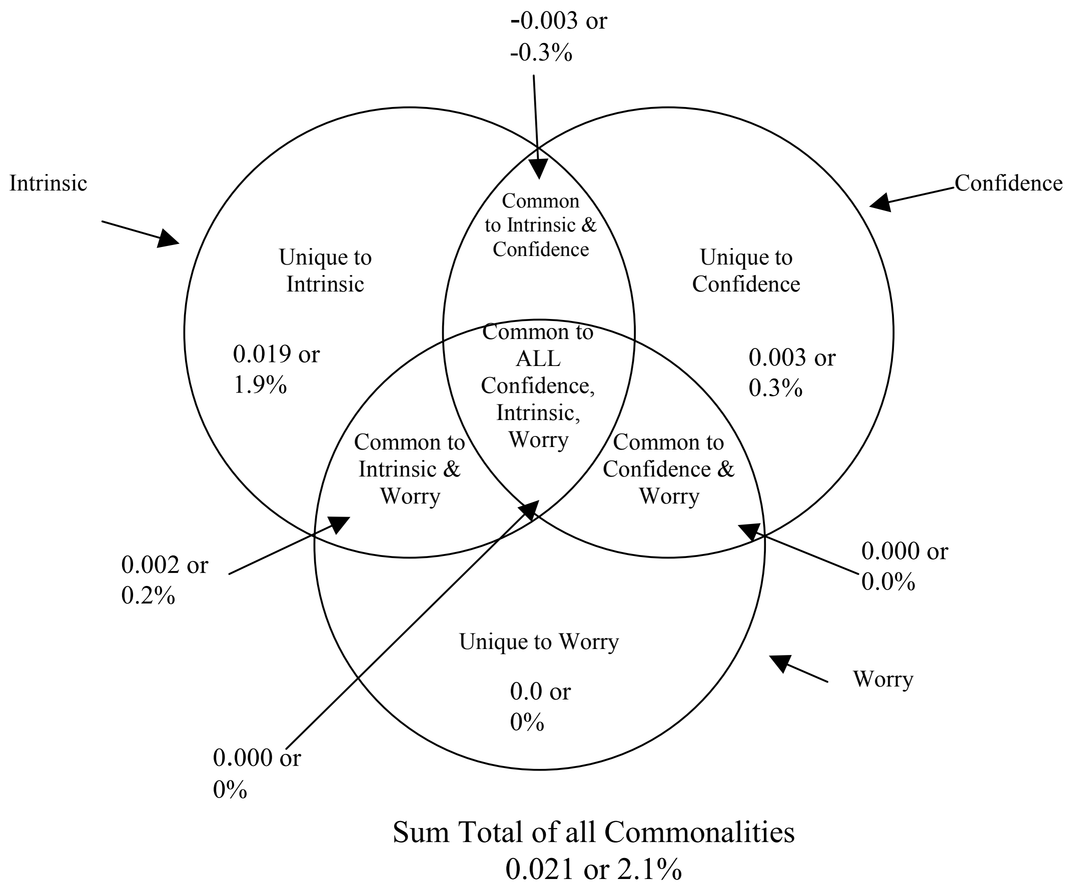

Figure 3. *Venn Diagram Showing Commonalities for Function II.*

In Function II the total variance explained is a paltry 2.1% This is hardly worthy of discussion except for the relatively large sample size to variable ratio and effect size originally indicated in the CCA. The effect size of 0.38, considered large in regards to educational research stands out in this case as well. The practical importance can not be neglected either. In review of other research on this topic, the effect size of 0.38 is large by comparison. The variance explained was partitioned into unique and common contributions and a few interesting observations are noticed.

On Function II (Figure 3) the results appear a little more interesting. Intrinsic value contributes the most variance explained 1.9% alone and confidence contributes 0.3% alone. When considering the common variance between confidence and intrinsic a -0.3% variance explained exists. This confounding seems to indicate that as the scores on confidence decreases (indicating less confidence) success on the criterion variables increase. In this case scale may influence the negative commonality. This interpretation defies logic and again implores the interpretation offered by Frederick (1999) that it should be interpreted as zero. Again, in Function II (Figure 3) worry, traditionally attributed as a major cause of poor performance in mathematics, was found to have virtually no influence.

Summary

After performing the CCA, sufficient evidence existed (i.e., an interpretable Rc^2) to continue and determine the unique and common contributions of the predictor variables. Particularly, the full model effect size of 0.38 aided the researcher in deciding to continue with further analysis. The CA yielded results on two functions. On Function I, the unique variance accounted for largely resides with the confidence variable (18.8%). This represents the overwhelming portion of the total variance 24.2% accounted for by all three of the variables - confidence, worry, and intrinsic value. This leads to an interesting supposition. First, contrary to contemporary findings this study seems to indicate that worry, contributing less than 1% of the variance, also referred to as math anxiety, is not a powerful predictor of mathematics achievement. Perhaps more time spent working on confidence and building "mathematics self-esteem" will improve mathematics achievement. Second, the results of Function II indicate that all three variables account for slightly more than 2.0% of the variance in the criterion set. This result is not very promising. However, of the variance accounted for intrinsic value accounts for 1.9 %, confidence accounts for 0.3%, and worry accounts for 0.0% of the total variance. On function II intrinsic value appears to be more helpful in predicting geometry achievement than either of the other two subscales. A list of all the SPSS syntax used in this analysis is listed in the Appendix.

The value of CA resides in the fact that the procedure yields unique and common variance explained from each of the predictor variables. The variance explained is not summative nor is it a result of interaction effects. The variance explained from the full model can be understood and the contributions of each separate variable can be interpreted in relation to the full model for the results of the unique effects. This helps to determine the most parsimonious model and relevant data sources, particularly when using a test containing subscales.

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Appendeix
SPSS Syntax for Conducting CA

Opens the file containing the data for the analyis

```
GET FILE
 "C:\WINDOWS\DESKTOP\Dissertation Data\Modified Dissertation Data File.sav"
 EXECUTE
```

Runs the descriptives that will be necessary for creating CRIT1 and CRIT2

```
 DESCRIPTIVES
   VARIABLES=spacerel gcksum
   /STATISTICS=MEAN STDDEV MIN MAX .
```

The full CCA syntax supplies the Rc² and the structure & function coefficients

```
Manova
 spacerel gcksum with int.val worry confid
 /print=signif(multiv eigen dimenr)
 /discrim(stan estim cor)alpha(.999))/design.
```

The syntax to create CRIT1 and CRIT2

```
COMPUTE crit1 = (.482*zspace)+(.645*zgck) .
 EXECUTE .
 COMPUTE crit2 = (-1.113*zspace)+(1.027*zgck) .
 EXECUTE .
```

All the syntax to run all possible combinations multiple regressions for the 3 predictor variables.

```
regression variables=crit1 crit2 int.val worry confid/
 dependent=crit1/enter int.val worry confid.
 regression variables=crit1 crit2 int.val worry confid/
 dependent=crit2/enter int.val worry confid.
regression variables=crit1 crit2 int.val worry confid/
 dependent=crit1/enter int.val confid.
regression variables=crit1 crit2 int.val worry confid/
 dependent=crit2/enter int.val confid.
regression variables=crit1 crit2 int.val worry confid/
 dependent=crit1/enter int.val worry.
regression variables=crit1 crit2 int.val worry confid/
 dependent=crit2/enter int.val worry.
regression variables=crit1 crit2 int.val worry confid/
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 dependent=crit2/enter int.val.
 regression variables=crit1 crit2 int.val worry confid/
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 dependent=crit2/enter confid.
regression variables=crit1 crit2 int.val worry confid/
 dependent=crit1/enter worry.
 regression variables=crit1 crit2 int.val worry confid/
 dependent=crit2/enter worry
```

Bigger is Not Better: Seeking Parsimony in Canonical Correlation Analysis via Variable Deletion Strategies

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This paper illustrates the value of applying the law of parsimony to canonical correlation analysis (CCA) solutions. The primary purpose of parsimony is that the more parsimonious the solution, the more replicable the model will be. The ultimate goal is to estimate an equal or reasonable amount of variance with the smallest variable set possible. A real-world data set is used that is composed of 287 sixth-grade students who were administered a geometry content knowledge test with three levels and a spatial visualization test as criterion variables, and a mathematics attitude survey with six subscales as predictor variables. Three different deletion methods are delineated in the paper that will assist the researcher in deleting predictor or criterion variables to obtain a more parsimonious canonical solution.

In research contents, the law of parsimony states that the fewer variables used to explain a situation, the more probable that the explanation will be closer to reality. In a canonical correlation analysis (CCA), Thorndike (1978) stated that "as the number of variables increase, the probable effect of these sources of error variation on the canonical correlation increases" (p. 188). This is because one source of sampling error comes from the number of measured variables. Therefore, as variable sets become more parsimonious there are greater probabilities that the results of the analysis will be replicable (Cantrell, 1999).

Rim (1972) suggested that models that are more parsimonious are not only more stable and replicable but also more generalizable. According to Thompson (1989), reducing the number of variables lessen Type II error probability since degrees of freedom model are also lessened. In an analysis with three criterion variables and six predictor variables, the 18 degrees of freedom would be reduced by nine if three predictor variables were deleted from the final model. Thompson (1984a) also suggested that dropping of variables in CCA would be synonymous with "backward elimination" stepwise procedures in multiple regression. Also purported was that this connection helped to reinforce the concept that all parametric techniques are subsumed under CCA as the classical form of the general linear model (Henson, 2000; Knapp, 1976). Therefore, the goal of a variable deletion strategy is to estimate as much variance with the smallest variable set possible. This paper will show that "bigger is not better", at least in reference to the number of variables, when using canonical correlation analysis.

Since Knapp (1978) demonstrated that canonical correlation analysis was the most general form of the general linear model, CCA has gained more in popularity. Thompson (1991) showed that CCA subsumes all other parametric methods including t-tests, point bisereal, ANOVA, regression, discriminant analysis, and MANOVA. CCA has been hibernating since Hotelling first developed the logic of CCA in 1936 more than 63 years ago. Besides Knapp's demonstration, computer statistical packages have made its use more easily accessible to researchers. As Pedhazur (1997) has noted, canonical correlation matrix computation can become "prohibitive" and "complex". Modern statistical packages almost eliminate the need to create these matrixes.

Because reality involves multiple effects and multiple effects have multiple causes, canonical analysis can more accurately represents this reality by explaining multiple relationships (Clark, 1975; Thompson, 1984a). Canonical correlation analysis appropriately examines the relationship between two sets of measured variables. An example would be comparing subtests of the WISC-R and the Woodcock Johnson that measure different intellectual abilities (Eastbrook, 1984). Multiple regression analysis could do the job f there were only one dependent varaible; however, canonical analysis goes a step farther by allowing multiple dependent variables. Furthermore, CCA maximizes a set of multiplicative weights all variables in the dependent and independent variable sets (Henson, 2000). Although it is not obvious, even in multiple regression a weight is developed for the dependent variable. However, since the dependent variable is not transformed to maximize some criterion, the weight is inescapably one (1).

Table 1. Initial Solution with Canonical Communality Coefficients Deletion Strategy #I

Variable Statistic	Function 1			Function 2			Function 3			
	Func.	r_s	r_s^2	Func.	r_s	r_s^2	Func.	r_s	r_s^2	h^2
spacerel.	-0.5	-0.845	71.40%	0.556	0.162	2.62%	0.956	0.509	25.91%	99.94%
level0	-0.179	-0.604	36.48%	1.008	0.510	26.01%	-0.617	-0.613	37.58%	100.07%
gcksum	-0.521	-0.901	81.18%	-1.197	-0.331	10.96%	-0.843	-0.279	7.78%	99.92%
Adequacy			63.02%			13.20%			23.76%	
Rd			16.13%			0.49%			0.45%	
Rc^2			25.60%			3.70%			1.9%	
*R*d			6.86%			0.68%			0.20%	
Adequacy			26.80%			18.35%			10.71%	
Useful	0.157	0.581	33.76%	0.153	-0.076	0.58%	-0.565	-0.463	21.44%	55.77%
Intrinsi	-0.096	0.426	18.15%	-0.579	-0.63	39.69%	-0.862	-0.571	32.60%	90.44%
Worry	-0.187	-0.081	0.66%	-0.829	-0.805	64.80%	0.531	0.292	8.53%	73.99%
Confid	0.932	0.972	94.48%	-0.023	-0.207	4.28%	0.787	0.083	0.69%	99.45%
Percep	0.046	0.244	5.95%	0.2	-0.061	0.37%	0.145	0.033	0.11%	6.43%
Success	0.061	0.279	7.78%	0.229	-0.061	0.37%	-0.222	-0.096	0.92%	9.08%

This present paper will illustrate three variable deletion strategies in CCA to yield the most parsimonious variable set. Parsimony will be sought for the predictor variable set, students' attitude toward mathematics, as opposed to the criterion variables, students' geometric and spatial visualization abilities. However, the same procedures could be applied to the criterion variable set.

The current data set comes from a study of 287 sixth-grade students from a south central state who were administered three tests. The *Mathematics Attitude Survey* (MATS) (Gierl & Bisanz, 1997), a Likert–type instrument, consisted of the six subscales of usefulness, intrinsic value, worry, confidence, perceptions, and attitude toward success. The six subscales served as the predictor set. The *Space Relations Portion* of the *Differential Aptitude Test* (Bennett, Seashore, & Wesman, 1973) assessed students' spatial sense focusing on visualization. The *Geometry Content Knowledge Test* (Carroll, 1998) was used to assess geometric content knowledge and to assign van Hiele (1984) geometry levels ranging from level 0 to level 2. The preceding two mathematics tests along with level 0 of the geometry content knowledge test served as the three criterion variables (spacerel, level 0, gcksum) in the study. The six subscales of the attitude survey (useful, intrinsi, worry, confid, percep, and success) served as the predictor set. The Statistical Package for the Social Sciences (SPSS) command syntax for running the CCA analysis was:

```
MANOVA
SPACEREL LEVEL0 GCKSUM WITH
USEFUL INTRINSI WORRY CONFID PERCEP SUCCESS
/PRINT=SIGNIF (MULTIV EIGEN DIMENR)
/DISCRIM (STAN ESTIM COR) ALPHA (.999)) / DESIGN.
```

The results of the analysis are compiled in Table 1, which is the suggested format for reporting canonical results.

According to Humphries-Wadsworth (1998), canonical correlation analysis is a "rich tool for examining the multiple dimensions of the synthetic variable relationships" (*p*. 6). In addition to the standardized function coefficients and structure coefficients, three other coefficients are often examined and can facilitate interpretation: canonical communality coefficients, canonical adequacy coefficients, and canonical redundancy coefficients (however, see Robert [1999] for discussion of the inadequacies of redundancy coefficients.

The researcher will now attempt to develop a clear process for completing the table. The "Func" (canonical function coefficient), the " r_s" (canonical structure coefficient) along with the Rc^2 (squared canonical correlation coefficient) for each function was obtained directly from the SPSS printout. The r_s^2 (squared canonical structure coefficient) was calculated by squaring the canonical structure coefficients for each variable and converting them into percentage format. The h^2 (communality coefficient) for each variable was obtained by summing all the r_s^2s. The adequacy coefficient, "how will a canonical variate represents the variance of the original variables in a domain" (Thompson. 1980, *p*.10), was an average of all the squared structure coefficients for the variables in one set with respect to one function. The adequacy coefficient for the criterion variable set was calculated by adding all the structure coefficients in the criterion set and dividing by the number of variables in the set and converting it into percentage format. The adequacy coefficient for the predictor set was determined by the same method. The redundancy coefficient, the redundancy of *C* (criterion variable set) given *P* (predictor variable set), was calculated by multiplying the adequacy coefficient by the Rc^2 for each function (Roberts, 1999).

After examining the full canonical analysis, the law of parsimony (Thorndike, 1978) can be invoked through a process called variable deletion. Various researchers (Cantrell, 1999; Rim, 1972; Stephens, 1996; & Thompson, 1984b) discussed approaches to achieve the most parsimonious variable set. This researcher will attempt to make the deletion process as understandable as possible. Three different strategies will be examined.

Variable Deletion

During the deletion process three coefficients will be consulted:

r_s^2 - squared canonical structure coefficient - how much variance a variable linearly shares with a canonical variate (Thompson, 1980).

$\mathbf{h^2}$ – canonical communality coefficients - sum of all $r_s^{\mathbf{2}}$; how much of the variance in a given observed variable is reproduced by the complete canonical solution (Thompson, 1991).

$Rc^{\mathbf{2}}$ - squared canonical coefficient– how much each function is contributing to the overall canonical solution (Thompson, 1991).

Variable Deletion Strategy #1

Deletion Strategy #1 looked at the h^2s only. The process involved the following steps:

1. Look at all the h^2s
2. Find the lowest h^2 and delete the corresponding variable
3. Rerun the CCA and recalculate the h^2s
4. Check the change to the Rc^2 for each function
5. If there is little change to Rc^2 find the next lowest h^2
6. Delete the variable with the corresponding lowest h^2 and repeat the process until the Rc^2 change is too great by researcher judgment.

Looking at Table 1, the predictor variables with the lowest h^2s were perceptions (6.34%) and success (9.08%). Both of these variables were quite a bit lower than the other four-predictor variables that ranged from 55.77 % to 99.45%. Through variable deletion strategy #1, the variable with the lowest h^2, perceptions, was dropped first. Table 2 showed the canonical analysis after perceptions was dropped. The Rc^2s were then examined for each function and there was only a very slight change. Function 1 did not change, Function 2 went from 3.7% to 3.6 %, and Function 3 remained the same. The Rc^2 change was less than 0.2% for only one function.

The remaining canonical solution still contained success with a h^2 of 9.0%. That variable was considerably lower than the other variables in Table 2, therefore, success was dropped and little change (less than 0.2%) was seen in the Rc^2s of each function as shown in Table 3. Function 1 changed from 25.6% to 25.5%, Function 2 changed from 3.6 % to 3.4%, and Function 3 changed from 1.9% to 1.8%. The limitations to this strategy involved the contributions that were not evaluated until after the variable was dropped. This could have caused keeping a large h^2 that only happened on the last canonical function and had a small Rc^2 effect size. Despite these limitations, the goal of parsimony was achieved by removing the two variables and only a very small change was noted in either the communality coefficients or the squared canonical coefficients of each function.

Table 2. *Canonical Solution After Dropping Perceptions Based on Canonical Communality Coefficients Deletion Strategy #I, Iteration #2*

Variable Statistic	Function 1			Function 2			Function 3			
	Func.	r_s	r_s^2	Func.	r_s	r_s^2	Func.	r_s	r_s^2	h^2
spacerel.	-0.503	-0.846	71.57%	0.522	0.142	2.02%	0.974	0.513	26.32%	99.90%
level0	-0.181	-0.605	36.60%	1.028	0.528	27.88%	-0.583	-0.596	35.52%	100.00%
gcksum	-0.516	-0.9	81.00%	-1.181	-0.324	10.50%	-0.524	-0.292	8.53%	100.02%
Adequacy			63.06%			13.46%			23.45%	
Rd			16.14%			0.48%			0.45%	
Rc^2			25.60%			3.6%			1.9%	
*R*d			6.62%			0.67 %			0.21%	
Adequacy			25.85%			18.60%			11.04%	
Useful	0.167	0.581	33.76%	0.211	-0.061	0.37%	-0.53	-0.467	21.81%	55.94%
Intrinsi	-0.093	0.427	18.23%	-0.56	-0.622	38.69%	-0.891	-0.603	36.36%	93.28%
Worry	-0.177	-0.08	0.64%	-0.817	-0.825	68.06%	0.525	0.255	6.50%	75.21%
Confid	0.934	0.973	94.67%	-0.03	-0.204	4.16%	0.802	0.079	0.62%	99.46%
Percep	0	0	0.00%	0	0	0.00%	0	0	0.00%	0.00%
Success	0.072	0.279	7.78%	0.286	-0.057	0.32%	-0.176	-0.098	0.96%	9.07%

Table 3. *Final Canonical Solution After Dropping Perceptions and Success Based on Communality Coefficients Deletion Strategy #I, Iteration #3*

Variable Statistic	Function 1			Function 2			Function 3			
	Func.	r_s	r_s^2	Func.	r_s	r_s^2	Func.	r_s	r_s^2	h^2
spacerel.	-0.504	-0.846	71.57%	0.583	0.171	2.92%	0.938	0.505	25.50%	100.00%
level0	-0.190	-0.610	37.21%	0.984	0.482	23.23%	-0.651	-0.628	39.44%	99.88%
gcksum	-0.509	-0.898	80.64%	-1.218	-0.349	12.18%	-0.441	-0.266	7.08%	99.90%
Adequacy			63.14%			12.78%			24.01%	
Rd			16.10%			0.43%			0.43%	
Rc^2			25.50%			3.40%			1.80%	
*R*d			6.29%			0.67%			0.20%	
Adequacy			24.67%			19.78%			10.97%	
Useful	0.175	0.582	33.87%	0.229	-0.075	0.56%	-0.584	-0.475	22.56%	57.00%
Intrinsi	-0.093	0.43	18.49%	-0.629	-0.664	44.09%	-0.845	-0.557	31.02%	93.60%
Worry	-0.153	-0.078	0.61%	-0.732	-0.840	70.56%	0.549	0.339	11.49%	82.66%
Confid	0.950	0.975	95.06%	0.082	-0.187	3.50%	0.764	0.087	0.76%	99.32%
Percep	0	0	0.00%	0	0	0.00%	0	0	0.00%	0.00%
Success	0	0	0.00%	0	0	0.00%	0	0	0.00%	0.00%

Variable Deletion #2

Deletion Strategy #2 looks at the contribution of each function the total canonical solution. The steps in the process are as follows:

1. Run a full CCA and look at the Rc^2 for each function.
2. Omit the function with the smallest Rc^2
3. Compute the subset of h^2s
4. Now find variable that has the lowest h^2; drop it from the original solution
5. Repeat the process until the remaining variables are reasonably close in their subset h^2 values. This will be a matter of researcher judgment.

The researcher employed strategy #2 in order to consider the value of each function to the whole canonical solution. Looking at Table 1, the lowest squared canonical coefficient (Rc^2) was found in Function 3 (1.9%), thus the entire function was dropped (Table 4). Note that the h^2 still showed that the

Table 4. *InitialCanonical Solution After Dropping Function 3 with Subset Canonical Communality Coefficients Deletion Strategy #2, Iteration #1*

Variable Statistic	Function 1			Function 2			
	Func.	r_s	r_s^2	Func.	r_s	r_s^2	h^2
spacerel.	-0.5	-0.845	71.40%	0.556	0.162	2.62%	74.03%
level0	-0.179	-0.604	36.48%	1.008	0.510	26.01%	62.49%
gcksum	-0.521	-0.901	81.18%	-1.197	-0.331	10.96%	92.14%
Adequacy			63.02%			13.20%	
Rd			16.13%			0.49%	
Rc^2			25.60%			3.70%	
*R*d			6.86%			0.68%	
Adequacy			26.80%			18.35%	
Useful	0.157	0.581	33.76%	0.153	-0.076	0.58%	34.33%
Intrinsi	-0.096	0.426	18.15%	-0.579	-0.63	39.69%	57.84%
Worry	-0.187	-0.081	0.66%	-0.829	-0.805	64.80%	65.46%
Confid	0.932	0.972	94.48%	-0.023	-0.207	4.28%	98.76%
Percep	0.046	0.244	5.95%	0.2	-0.061	0.37%	6.33%
Success	0.061	0.279	7.78%	0.229	-0.061	0.37%	8.16%

Table 5. *Canonical Solution After Dropping Perceptions Based on Subset Canonical Communality Coefficients Deletion Strategy #2, Iteration #2*

Variable Statistic	Function 1			Function 2			
	Func.	r_s	r_s^2	Func.	r_s	r_s^2	h^2
spacerel.	-0.503	-0.846	71.57%	0.522	0.142	2.02%	73.59%
level0	-0.181	-0.605	36.60%	1.028	0.528	27.88%	64.48%
gcksum	-0.516	-0.9	81.00%	-1.181	-0.324	10.50%	91.50%
Adequacy			63.06%			13.46%	
Rd			16.14%			0.48%	
Rc^2			25.60%			3.60%	
*R*d			6.62%			0.67%	
Adequacy			25.85%			18.60%	
Useful	0.167	0.581	33.76%	0.211	-0.061	0.37%	34.13%
Intrinsi	-0.093	0.427	18.23%	-0.56	-0.622	38.69%	56.92%
Worry	-0.177	-0.08	0.64%	-0.817	-0.825	68.06%	68.70%
Confid	0.934	0.973	94.67%	-0.03	-0.204	4.16%	98.83%
Percep	0	0	0.00%	0	0	0.00%	0.00%
Success	0.072	0.279	7.78%	-0.286	-0.057	0.32%	8.11%

Function 3 (1.9%), thus the entire function was dropped (Table 4). Note that the h^2 still showed that the variables of perception and success had the lowest h^2s, 6.33% and 8.16% respectively. Perceptions was first variable deleted and the results of the canonical solution was displayed in Table 5. Table 6 indicated an even more parsimonious solution after dropping success. Since a subset with a close grouped $\underline{h^2}$ subset was sought, this researcher also dropped useful (34.43%). Table 7 showed the smallest set of variables with a relatively close range of communality coefficients. The h^2s were intrinsic (69.1%), worry (67.28%), and confidence (99.79%). Based on the literature and researcher judgment, the iteration process was ended. Of the three remaining variables, worry had a squared structure coefficient of .53% on Function1 but a 66.75% on Function 2. Reverse effects were seen for confidence that had a r_s^2 of 97.42% on Function1 but 2.37% on Function 2.One limitation of strategy #2 was that it did not consider functions with small Rc^2 values. In addition, the variations as to where h^2 values came from as shown in worry and confidence were not considered.

Table 6. *Canonical Solution After Dropping Perceptions and Success Based on Subset Canonical Communality Coefficients Deletion Strategy #2, Iteration #3*

Variable Statistic	Function 1			Function 2			h^2
	Func.	r_s	r_s^2	Func.	r_s	r_s^2	
spacerel.	-0.504	-0.846	71.57%	0.583	0.171	2.92%	74.50%
level0	-0.190	-0.610	37.21%	0.984	0.482	23.23%	60.44%
gcksum	-0.509	-0.898	80.64%	-1.218	-0.349	12.18%	92.82%
Adequacy			63.14%			12.78%	
Rd			16.10%			0.43%	
Rc^2			25.50%			3.40%	
*R*d			6.29%			0.67%	
Adequacy			24.67%			19.78%	
Useful	0.175	0.582	33.87%	0.229	-0.075	0.56%	34.43%
Intrinsi	-0.093	0.430	18.49%	-0.629	-0.664	44.09%	62.58%
Worry	-0.153	-0.078	0.61%	-0.732	-0.840	70.56%	71.17%
Confid	0.950	0.975	95.06%	0.082	-0.187	3.50%	98.56%
Percep	0	0	0.00%	0	0	0.00%	0.00%
Success	0	0	0.00%	0	0	0.00%	0.00%

Table 7. *Final Canonical Solution After Dropping Perceptions, Success and Useful Based on Canonical Communality Coefficients Deletion Strategy #2, Iteration #4*

Variable Statistic	Function 1			Function 2			h^2
	Func.	r_s	r_s^2	Func.	r_s	r_s^2	
spacerel.	-0.491	-0.837	70.06%	0.692	0.225	5.06%	75.12%
level0	-0.216	-0.629	39.56%	0.892	0.393	15.44%	55.01%
gcksum	-0.503	-0.9	81.00%	-1.268	-0.389	15.13%	96.13%
Adequacy			63.54%			11.88%	
Rd			16.20%			0.40%	
Rc^2			25.50%			3.40%	
*R*d			4.96%			0.68%	
Adequacy			19.46%			19.90%	
Useful	0	0	0.00%	0	0	0.00%	0.00%
Intrinsi	-0.065	0.434	18.84%	-0.682	-0.709	50.27%	69.10%
Worry	-0.139	-0.073	0.53%	-0.68	-0.817	66.75%	67.28%
Confid	1.031	0.987	97.42%	0.249	-0.154	2.37%	99.79%
Percep	0	0	0.00%	0	0	0.00%	0.00%
Success	0	0	0.00%	0	0	0.00%	0.00%

Variable Deletion #3

Deletion Strategy # 3 considered weighted h^2. This strategy looked at the variables' contribution to the complete canonical solution. The steps were as follows:

1. Multiply Rc^2 by each r_s^2 and add the products together for each function to obtain the weighted h^2 for each variable.
2. Drop the lowest weighted h^2, repeat the previous step.
3. Look at the change in Rc^2; if there is little change, drop the variable with the next lowest h^2.
4. Take out as many variables as possible without compromising the Rc^2.

In order to consider the limitations of variable deletion #2, the weighted communality coefficients helped the researcher obtain a more realistic view of how much each predictor variable contributes to the total canonical analysis. Using the above algorithm in step 1, the weighted communality coefficients

Table 8. *Initial Canonical Solution with Weighted Canonical Communality Coefficients Deletion Strategy #3, Iteration #1*

Variable Statistic	Function 1			Function 2			Function 3			Weighted
	Func.	r_s	r_s^2	Func.	r_s	r_s^2	Func.	r_s	r_s^2	h^2
spacerel.	-0.500	-0.845	71.40%	0.556	0.162	2.62%	0.956	0.509	25.91%	18.87%
level0	-0.179	-0.604	36.48%	1.008	0.510	26.01%	-0.617	-0.613	37.58%	11.02%
gcksum	-0.521	-0.901	81.18%	-1.197	-0.331	10.96%	-0.843	-0.279	7.78%	21.34%
Adequacy			63.02%			13.20%			23.76%	
Rd			16.13%			0.49%			0.45 %	
Rc^2			25.60%			3.70%			1.9%	
*R*d			6.86%			0.68%			0.20%	
Adequacy			26.80%			18.35%			10.71%	
Useful	0.157	0.581	33.76%	0.153	-0.076	0.58%	-0.565	-0.463	21.44%	9.07%
Intrinsi	-0.096	0.426	18.15%	-0.579	-0.63	39.69%	-0.862	-0.571	32.60%	6.73%
Worry	-0.187	-0.081	0.66%	-0.829	-0.805	64.80%	0.531	0.292	8.53%	2.73%
Confid	0.932	0.972	94.48%	-0.023	-0.207	4.28%	0.787	0.083	0.69%	24.36%
Percep	0.046	0.244	5.95%	0.2	-0.061	0.37%	0.145	0.033	0.11%	1.54%
Success	0.061	0.279	7.78%	0.229	-0.061	0.37%	-0.222	-0.096	0.92%	2.02%

Table 9. *Canonical Solution with Canonical Weighted Communality Coefficients After Dropping Perceptions Deletion Strategy #3, Iteration 2*

Variable Statistic	Function 1			Function 2			Function 3			Weighted
	Func.	r_s	r_s^2	Func.	r_s	r_s^2	Func.	r_s	r_s^2	h^2
spacerel.	-0.503	-0.846	71.57%	0.522	0.142	2.02%	0.974	0.513	26.32%	18.89%
level0	-0.181	-0.605	36.60%	1.028	0.528	27.88%	-0.583	-0.596	35.52%	11.05%
gcksum	-0.516	-0.900	81.00%	-1.181	-0.324	10.50%	-0.524	-0.292	8.53%	21.28%
Adequacy			63.06%			13.46%			23.45%	
Rd			16.14%			0.48%			0.45%	
Rc^2			25.60%			3.60%			1.90%	
*R*d			6.62%			0.67%			0.21%	
Adequacy			25.85%			18.60%			11.04%	
Useful	0.167	0.581	33.76%	0.211	-0.061	0.37%	-0.530	-0.467	21.81%	9.07%
Intrinsi	-0.093	0.427	18.23%	-0.560	-0.622	38.69%	-0.891	-0.603	36.36%	6.75%
Worry	-0.177	-0.080	0.64%	-0.817	-0.825	68.06%	0.525	0.255	6.50%	2.74%
Confid	0.934	0.973	94.67%	-0.030	-0.204	4.16%	0.802	0.079	0.62%	24.40%
Percep	0	0	0.00%	0	0	0.00%	0	0	0.00%	0.00%
Success	0.072	0.279	7.78%	0.286	-0.057	0.32%	-0.176	-0.098	0.96%	2.02%

were obtained and examined. Table 8 illustrated the entire canonical solution showing weighted communality coefficients. Since the variable perceptions had the lowest weighted h^2 (1.54%), it was first dropped resulting in Table 9. The next lowest, success (2.02%), was then deleted resulting in Table 10. The next smallest weighted h^2 came from worry (2.91%), which was then deleted. The results are displayed in Table 11. After these three deletions from the canonical solution, the Rc^2 changes were small, 0.7% in Function 1, 1.3% in Function 2, and 1.4% in Function 3.

Since none of the variables remaining had their highest squared structure coefficient (r_s^2) in Function 3, which also had the lowest Rc^2 (0.5%), Function 3 was now dropped and the most parsimonious solution set resulted in two functions with three predictors displayed in Table 12. The researcher considered this the best combination of the deletion strategies since both the functions and the weighted h^2s were considered. The results indicated that when students consider mathematics useful and most importantly are confident in mathematics, they perform better on tests that measure their geometric content knowledge

Table 10. *Initial Solution with Canonical Weighted Communality Coefficients After Dropping Perceptions and Success Deletion Strategy #3, Iteration*

Variable Statistic	Function 1			Function 2			Function 3			Weighted h^2
	Func.	r_s	r_s^2	Func.	r_s	r_s^2	Func.	r_s	r_s^2	
spacerel.	-0.504	-0.846	71.57%	0.583	0.171	2.92%	0.938	0.505	25.50%	18.91%
level0	-0.190	-0.610	37.21%	0.984	0.482	23.23%	-0.651	-0.628	39.44%	11.11%
gcksum	-0.509	-0.898	80.64%	-1.218	-0.349	12.18%	-0.441	-0.266	7.08%	21.22%
Adequacy			63.14%			12.78%			24.01%	
Rd			16.16%			0.46%			0.46%	
Rc^2			25.60%			3.60%			1.90%	
*R*d			6.32%			0.71%			0.21%	
Adequacy			24.67%			19.78%			10.97%	
Useful	0.175	0.582	33.87%	0.229	-0.075	0.56%	-0.584	-0.475	22.56%	9.12%
Intrinsi	-0.093	0.43	18.49%	-0.629	-0.664	44.09%	-0.845	-0.557	31.02%	6.91%
Worry	-0.153	-0.078	0.61%	-0.732	-0.840	70.56%	0.549	0.339	11.49%	2.91%
Confid	0.950	0.975	95.06%	0.082	-0.187	3.50%	0.764	0.087	0.76%	24.48%
Percep	0	0	0.00%	0	0	0.00%	0	0	0.00%	0.00%
Success	0	0	0.00%	0	0	0.00%	0	0	0.00%	0.00%

Table 11. *Final Canonical Solution After Dropping Perceptions, Success, and Worry with Weighted Canonical Communality Coefficients Deletion Strategy 3, Iteration 4*

Variable Statistic	Function 1			Function 2			Function 3			Weighted h^2
	Func.	r_s	r_s^2	Func.	r_s	r_s^2	Func.	r_s	r_s^2	
spacerel.	-0.508	-0.841	70.73%	-1.038	0.439	19.27%	0.371	0.316	9.99%	18.12%
level0	-0.244	-0.643	41.34%	0.338	-0.041	0.17%	-1.12	-0.765	58.52%	10.59%
gcksum	-0.468	-0.889	79.03%	-1.228	-0.455	20.70%	0.459	0.056	0.31%	20.18%
Adequacy			63.70%			13.38%			22.94%	
Rd			15.86%			0.32%			0.11%	
Rc^2			24.90%			2.40%			0.50%	
*R*d			6.27%			0.35%			0.05%	
Adequacy			25.19%			14.50%			10.27%	
Useful	0.159	0.586	34.34%	-0.129	-0.294	8.64%	-1.15	-0.755	57.00%	9.04%
Intrinsi	-0.124	0.44	19.36%	-1.131	-0.883	77.97%	0.345	0.16	2.56%	6.70%
Worry	0	0	0.00%	0	0.000	0.00%	0	0	0.00%	0.00%
Confid	0.973	0.987	97.42%	0.581	-0.064	0.41%	0.529	0.144	2.07%	24.28%
Percep	0	0	0.00%	0	0	0.00%	0	0	0.00%	0.00%
Success	0	0	0.00%	0	0	0.00%	0	0	0.00%	0.00%

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and spatial visualization. Also, students who receive extrinsic rewards perform better than those students who rely on intrinsic motivation.

The goal of all these deletion strategies was a more parsimonious solution. Therefore, choosing the smaller variable set when the same amount of variance can be accounted for was achieved. Just remember "bigger is not better!" in canonical correlation analysis.

Table 12. *Final Canonical Solution with Combination of Variable Deletion Strategies With Weighted Canonical Communality Coefficients*

Variable Statistic	Function 1			Function 2			Weighted
	Func.	r_s	r_s^2	Func.	r_s	r_s^2	h^2
spacerel.	-0.508	-0.841	70.73%	-1.038	0.439	19.27%	18.07%
level0	-0.244	-0.643	41.34%	0.338	-0.041	0.17%	10.30%
gcksum	-0.468	-0.889	79.03%	-1.228	-0.455	20.70%	20.18%
Adequacy			63.70%			13.38%	
Rd			15.86%			0.32%	
Rc^2			24.90%			2.40%	
*R*d			6.27%			0.35%	
Adequacy			25.19%			14.50%	
Useful	0.159	0.586	34.34%	-0.129	-0.294	8.64%	8.76%
Intrinsi	-0.124	0.44	19.36%	-1.131	-0.883	77.97%	6.69%
Worry	0	0	0.00%	0	0.000	0.00%	0.00%
Confid	0.973	0.987	97.42%	0.581	-0.064	0.41%	24.27%
Percep	0	0	0.00%	0	0	0.00%	0.00%
Success	0	0	0.00%	0	0	0.00%	0.00%

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A Longitudinal Study of Familial Influences on Marijuana Use by Mexican American Middle School Students

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The purposes of this longitudinal parent-child investigation were to: (a) investigate the influence of familial factors (marital status of the husband and wife, family transience, adult cigarette smokers in the home, and parent-child communication style) on the use of marijuana among Mexican American middle school youth, and (b) use a growth curve model to estimate and examine the effect of time on the pattern and change rate of marijuana use among Mexican American school-age youth over a three year period. Methodologically, this was accomplished by applying a random-effects model in which student characteristics were construed as fixed effects at the micro-level and familial factors were treated as random effects at the macro-level in their relation to students' use of marijuana over a three year period. Results indicated that marijuana use increased across time among the students. Also, the quality of parent-child communication differentiated marijuana users from non-users. Gender of students, adults smoking cigarettes in the home, family transience, and divorce were all significantly related to substance use in the population studied.

Over the past several decades studies of adolescent substance use have focused on the prevalence, distribution, and use of illicit and licit substances among American children and youth (Johntson, O'Malley, & Bachman, 1996; U. S. Department of Health and Human Services, 1992). For instance, in 1997-1998 school year, 31% of 10th graders reported the use of marijuana while 9.7% of 8th graders also used it at least one time (The 1998 Monitoring the Future). Overall, 21% of 8th graders and 31% of 10th graders have used at least one illicit drug during that same period. Researchers have investigated a wide variety of individual psychological attributes, behaviors, demographic characteristics, genetic factors, and environmental influences on adolescents that they have classified as either risk or protective factors for involvement in the use of alcohol and other substances (Bry, McKeon & Pandina, 1982; Hawkins & Catalano, 1989; Jesser & Jesser, 1977; Kandel, 1978, Kandel & Foust, 1975; Newcomb & Bentler, 1988; Newcomb, Maddahian, & Bentler, 1986; Vega, Zimmerman, Warheit, Apospori, & Gil, 1992).

For example, in the area of demographic characteristics the vast majority of national and state surveys find that males engage in illicit substance use at an earlier age and more frequently when compared to their female counterparts (Gilbert & Cervantes, 1986; Murray, Perry, O'Connell, & Schmid, 1987; Newcomb, Maddahian, Skager, & Bentler, 1987; United States Department of Health and Human Services, 1992). Studies have also found that divorce within the family acts as a risk factor for substance use for female, as well as male children (Cadoret, Troughton, O'Gorman, Heywood, 1986). In other studies, school related problems, psychological problems, and the inter- and intrapersonal stress that accompanies residential transience, or frequency of family mobility have emerged as risk factors for youth who engage in alcohol and other substance use (Humke & Schaefer, 1995; Puskar & Martsolf, 1994).

Much of the research in this area specifically examines parental and familial influences on adolescents' substance use by soliciting youngsters perceptions of their parents' beliefs and/or behaviors on issues related to substance use, specific parental style in which they were reared, and other family variables. For instance, studies have found that youngsters who were heavy users of substances felt more rejected by their parents, experienced less emotional warmth from their parents, and rated their parents' rearing behavior as more overprotective than non-user control subjects (Emmelkamp & Heeres, 1988). Other researchers have found parental and familial variables that influence the substance using behaviors of adolescents to include cigarette smoking and other substance use among one or both parents (Brook, Whiteman, Gordon, & Brook, 1985; Kandel, 1990; McDermott, 1984), disciplinary problems in the home, an overly restrictive discipline style and maternal rejection (Vicary & Lerner, 1986).

In a retrospective case study approach Low and Sibley (1991) asked 17 adults identified as problem drinkers details about their past home life and parent interactions and relationships as adolescents. Results indicated that extreme modes of control, such as highly strict rules and discipline enforcement or households with no clear rules at all, were significant influences to their problem drinking. Cooper and Olson (1977) surveyed adolescents and found low perceived parental support was associated with substance use. A number of other studies (Coombs, Paulson, & Richardson, 1991; Elliot, Huizinga, & Ageton, 1982; Halebsky, 1987; Hawkins, Lishner, & Catalano, 1985; Jurich, Polson, Jurich, & Bates, 1985; Jurich & Polson, 1984; Prendergast, 1974; Vicary & Lerner, 1986; Wills, Vaccaro, & McNamara, 1992) found positive relationships, including the multifaceted aspects of positive parent-child interactions and general parental communicative style (verbal and non-verbal) with children works as a protective, or resilience factor against youthful involvement in substance use.

Unfortunately, fewer studies have actually involved parents as direct sources of obtaining information on parental and familial influences as they relate to adolescent substance use (Brook, Whiteman, Cohen, Shapiro, & Balka, 1995; Shedler & Block, 1990). Studies using this approach have found that high levels of parental support, as well as positive adolescent-parent communication, are key elements in the prevention of alcohol and drug use and other deviant behaviors. Parental nurturance emerges from all of these studies as a key factor in preventing problem drinking and problem behaviors among adolescents (Barnes & Farrell, 1992; Barnes, 1984; Barnes, Farrell, & Banerjee, 1994). Kandel (1973) interviewed parents and youngsters and found that peer and parent influences on the use of substances is synergistic. The highest rates of marijuana use were observed among adolescents whose parents and friends used marijuana or other substances, leading to the notion that parental and peer modeling play a role in substance use.

Shedler and Block (1990) interviewed parents and their children and determined, among other things, that compared to mothers of substance experimenters, the mothers of frequent users could be described as hostile, not spontaneous with their children, not responsive or sensitive to their children's needs, critical of their children and rejecting of their ideas and suggestions, not supportive and encouraging of their children, cold, unresponsive, and unprotective. They appear to give their children little encouragement, while, conjointly, they were pressuring and overly interested in their children's "performance". All of these modes of interaction and communication were conducive to the adolescent substance use.

Prendergast and Thum (1973) and Prendergast (1977) found that alcohol use (in the first study) and marijuana use (in the second study) in adolescents was significantly correlated with the child's perceived style of communication with their father, particularly psychological tension. Wills, Vaccaro and McNamara (1992) and Barnes (1982) both found family support, including poor communication with parents was associated with adolescent use of licit and illicit substances. Gantman (1978) compared family interaction patterns within a number of families and found well-adjusted families (as opposed to families with emotionally disturbed and drug-abusing adolescents) displayed clearer communication among family members. This was also true in a study conducted by Lowe and Sibley (1991) in which it was found that "connected" patterns of family interactions which were characterized as a pattern of interaction suggesting good communication among family members had lower levels of adolescent substance use.

A limited number of studies have investigated the influence of family factors on the initiation and continued use of substances within the context of an intraethnic, all Latino population. Watts and Wright (1991) found that lack of family support, parental supervision, and/or parental drug use is significantly related to substance use among Latino youth. In another study (Gfroerer & De La Rosa, 1993) Latino youth and one parent were interviewed about family variables and their relationship to youngsters' substance use. The researchers found that substance use by mothers (particularly cigarette smoking) was highly correlated with substance use by their children.

Smith, Joe, and Simpson (1991) investigated parental influences on illicit substance use by Mexican American youth by interviewing both youth and their mothers on vital information pertaining to characteristics of parents of users, together with indicators of home environment and psychological status, in relation to their child's behavioral and emotional adjustment. Children of married mothers used fewer illicit substances in the first year after completing a Drug Prevention Program as opposed to children from divorced families, who evidenced continued substance use difficulties.

A research focus on parental communication style and other issues pertaining to family seems appropriate given the literature on Latinos in general and Mexican Americans in particular (Vasquez, 1998). Mexican American family members (including extended family) by tradition provide warmth and security for one another throughout their life (Griswold & del Castillo, 1984). This "familism" is one of the most important characteristic of "la familia" of Mexican Americans (Sena-Rivera, 1979; Ramirez & Arce, 1981) and has been described as a strong feeling of identification, dependence, loyalty, reciprocity, and solidarity among members of the family (Marin & Marin, 1991). This characteristic has been found to a greater extent among U.S. born Mexican Americans than among other ethnic and racial groups in the U.S. (See Ramirez & Arce, 1981 for a review). Strong familial support and positive communication (between children and parents, and extended family members) has been identified as a protective factor in stress resistant or resilient children (Garmezy, 1985; Kumpfer & Alvarado, 1995; Masten & Garmezy, 1985; Ramirez, 1980).

The purposes of this longitudinal parent-child investigation were to: (a) investigate the influence of familial factors (marital status of the husband and wife, family transience, adult cigarette smokers in the home, and parent-child communication style) on the use of marijuana among Mexican American youth, and (b) use a growth curve model to estimate and examine the effect of time on the pattern and change rate of marijuana use among Mexican American school-age youth over a three year period. Methodologically, this was accomplished by applying a random-effects model in which students' characteristics were construed as fixed effects at micro-level and familial factors were treated as random effects at macro-level in their relation to students' substance use over a period of three years.

Method

Subjects and Data Construction

Data used for this investigation was extracted from a longitudinal study in South Central Texas. Students in middle school, grade 6, 7, and 8 were surveyed regarding their use of substances, as well as on a set of psychological and social measures during three consecutive years. During the second year of the study, 720 students were randomly selected to have their parents participate in an interview protocol. These families were contacted by trained university students using telephone numbers provided by the school district. Three hundred and ninety-three families were successfully contacted and subsequently participated in the family interviews. Forty-one interviews were completed improperly (unmatchable cases and incomplete surveys) and were not usable for data analysis. The remainder of the families did not participated in the study due to difficulty contacting the parents (disconnected phone service, incorrect phone numbers, incorrect addresses or difficulty arranging an interview due to both parents working). Only a small portion of the families contacted refused to participate in the study. Therefore, the overall successful rate of parental interviews was 49%. A moderate completion rate was expected given the transient nature of residents of the community.

Inclusion of subjects in this study was based on the two criteria. First, a student's family must have been interviewed at year two of the study, and secondly, each student must have participated in at least two of the three yearly in-school surveys. Procedure and justification for dealing with incomplete cases will be discussed later in this article.

Procedure

Parental consent for students' participation in the school-based longitudinal survey study was established by both mailing a consent card with return postage to each student's family address and sending home a consent card. Student's parents or guardians were told about the nature of survey and were requested to indicate their willingness to have their child participate in the study. Prior to the survey administration, students were informed that their participation was voluntary and they could terminate their participation at any point during the survey. The overall consent rate of parents was 95%. The survey was administered to the students in school within intact classes by trained university students. Each question and choice on the survey was read aloud in English to control for readability of the survey.

The student survey dealt with questions pertaining to ethnicity, gender, and a Substance Use Inventory in which information was sought from each student as to their use of marijuana over the past year. The students were paid $1.00 after the survey for their participation.

The family interview was conducted by trained bilingual (English-Spanish) university students over a period of three months. Following a standardized protocol, interview staff first made at least three attempts to contact a family via telephone (86% of the interviews were conducted by way of telephone and 14% of the interviews were conducted in the home of the parents in face-to-face interviews; 74% of the interviews were conducted in English, 20% were conducted in Spanish, and 6% were conducted in both English and Spanish). During the last month of the interview, all families not contacted by telephone were visited by a pair of interviewers using addresses provided in school records. After the families agreed to participate in the study, the interview staff administered a closed-ended questionnaire in the language that the parents felt most comfortable with at their home. Interviews were conducted with the female head of the household (the mother in the majority of cases) without the presence of the student or any adult in the room.

Measures

Variables used in the present study were collected through both student and parent interviews. They are described below:

Self-reported use of marijuana. Student were asked to indicate how many times he or she had smoked marijuana in the last year. The response scale consisted of none, 1-2 times, 3 or more times. Marijuana use was coded as "no" if a student reported no use in last year and as "yes" if he or she reported at least 1-2 times of use2 .

Student's gender. Students reported their gender. Responses were coded as "F" for female and "M" for male.

Parental marital status. Parental marital status was coded as either "married" or "not married" (divorced or separated). This information was collected during the family interview.

Family residential transience. Parents reported the number of times the family moved residences in last three years was gathered during family interviews. Family transience was coded as "stable" if they had not moved and as "unstable" if it had moved residences at least once in the last three years.

Parent-child communication. Parents reported information for The Open Communication subscale of the "Parental Support Scale" detailing the level (quality and quantity) of information exchange between parents and child (Barnes & Olson, 1982). Adult respondents were asked about the communication process in the family. Respondent's score were divided into two groups (free communication and problems in communication) using a median split.

Data Analysis

Logistic regression was first applied to examine the cross-sectional relationships between marijuana use by students and its covariates at each time point. This would help to uncover the complicated relationships among the covariate variables. Conditional odd ratios (ORs) and 95% confidence intervals (CIs) were calculated for the covariate variables.

A multivariate hierarchical linear model (HLM) was used to study the growth curve of marijuana use in this longitudinal data set. HLM estimates individual parameters that describe how particular individuals change over time. Individual changes are estimated based on data from a previous timepoint which lead to more common overall population trends. HLM is more advantageous compared to the traditional regression approach (see Bryke & Ruadenbush, 1987; Hedeker & Gibbons, 1996; Goldstein, 1995 for a more detailed description of HLM).

First, error terms can be flexibly specified and treated as fixed, randomly varying, and non-randomly varying at each level of the estimation equations in HLM. Second, HLM avoids weaknesses in the repeated measure design in traditional longitudinal studies that only focus the final data point alone and ignore changes in covariates between initial and final timepoints.

Third, HLM does not require subjects to be measured at the same number of timepoints and therefore allows subjects with incomplete data across timepoints to be included in the analysis. The analysis is based on the available repeated observations on which subjects have data. Therefore, the analysis is more powerful and avoids selection biases because it includes all available subjects. Furthermore, HLM permits the use of different types of covariates to model the change in dependent variable due to both stable/invariant characteristics (e.g., their gender and their parents' level of acculturation), and unstable/time-varying characteristics (e.g., self-esteem and association with deviant peers). Finally, in contrast to the traditional approaches to longitudinal studies, HLM can estimate average change (across time) in a population as well as individual change for each subject. It provides a more realistic description of behavior change by considering different trends of each individual.

The growth curve of marijuana use was modeled by estimating linear and acceleration rates of substance use at level 1 as well as estimating randomly-varying effects of familial factors at level 2. Conceptually, the level 1 model represented the traditional regression models in which linear and quadratic trends of changes in dependent variable across three time points were assessed with the exception that the error term was refined as a combination of independent errors and random effects associated with the cluster (i.e. individual) effect. At level 2, the model was constructed to identify specific contributions of contextual variables (i.e. familial factors) and random effects due to cluster effect on level 1 parameter estimates. Since this was a two-level model, the error term at level 2 model was treated as a fixed term.

Specification of parameter estimates for the present study are as the following:

Level 1 Model

$$Y_{it} = \pi_{0i} + \pi_{1i}\,\alpha_{it} + \pi_{2i}\,\alpha_{2it} + \varepsilon_{it} \quad \varepsilon_{it} \sim N(0, \sigma^2)\ ,$$

whereY_{it} is the marijuana use index for student i at year t, t = 1, ..., 3; i = 1, ..., 295; α_{it} = year of survey - 1 so that $\alpha_{it} = 0$ at year 1 of the survey; α_{2it} = year of survey * year of survey so that α_{2it} represents the quadratic term to measure the acceleration rate π_{0i} is therefore the expected level of marijuana use at year 1 of the survey for student i; π_{1i} is therefore the expected rate of change of marijuana use per year of the survey for student I; π_{2i} is therefore the expected acceleration of change rate of marijuana use per year of the survey for student i; ε_{it} is a random error

Level 2 Model

$$p_{0i} = b_{00} + X_{0k} + m_{0i}$$
$$p_{1i} = b_{10} + m_{1i}$$
$$p_{2i} = b_{20} + m_{1i}\ ,$$

where b_{00} is the population mean of marijuana use index at year 1 of the survey; X_{0k} is the random-varying covariate, k= 1,..., 5 (parental marital status, parental-child communication, student's gender, family transience, gender and marital status interaction); b_{0k} is the fixed effect of random-varying covariate X_{0k}; b_{10} is the population mean rate of change of marijuana use index; b_{20} is the population acceleration rate of change of marijuana use index; m_{0j}, m_{1j}, m_{2j}, are random effects associated with student i and assumed N(0, τ2).

Results

Multivariate Logistic Analysis of Familial Variables

Table 1 and Table 2 show the univariate statistics and results of logistic regression on marijuana use at each time point. For year 1, males, poor parent-child communication, a cigarette smoking adult in the home, and females in a divorced household seemed to be related to more reported use of marijuana by the adolescents, while at year 2 students who were male, had poor parent-child communication, had a cigarette smoking adult in the home, and were female living in a divorced home reported more marijuana

Table 1. Descriptive statistics of dependent and level-2 measures

		Used Marijuana in Year 1	Used Marijuana in Year 2	Used Marijuana in Year 3
Parental Marriage Status	Married	47 (27.0%)	82 (39.0%)	71 (43.6%)
	Not Married	13 (36.1%)	25 (48.1%)	16 (51.6%)
Gender	Male	40 (36.7%)	65 (50.4%)	52 (56.5%)
	Female	20 (19.8%)	42 (31.6%)	35 (34.3%)
Parental-child Communication	Open	27 (29.0%)	40 (34.8%)	37 (44.0%)
	Closed	33 (28.2%)	67 (45.6%)	50 (45.5%)
Smokers living in smoking	None	32 (23.5%)	66 (37.9%)	54 (41.9%)
	Yes	28 (37.8%)	41 (46.6%)	33 (50.8%)
Moved in the last three years	No	49 (28.0%)	89 (41.8%)	70 (43.8%)
	Yes	11 (31.4%)	18 (36.7%)	17 (50.0%)

Table 2. Logistic regression on marijuana use at each time point.

	Marijuana Use at Year 1.			Marijuana Use at Year 2.			Marijuana Use at Year 3.		
Variable	B	SE	OR	B	SE	OR	B	SE	OR
MAR1_95(1)	.0370	.5271	1.0377	-.0983	.4398	.9064	.1894	.5788	1.2086
GENDER(1)	-1.1015	.3718	.3324 [b]	-1.0034	.2961	.3666 [a]	-1.0393	.3328	.3537 [d]
COMMU2(1)	-.0964	.3227	.9081	.4598	.2662	1.5838 [a]	.0435	.3044	1.0445
SMOKER(1)	.6850	.3254	1.9838 [b]	.4013	.2760	1.4938	.4310	.3222	1.5388
MOVING(1)	.2233	.4278	1.2502	-.2499	.3447	.7788	.4026	.4067	1.4957
INT_1	1.1206	.8021	3.0666	1.0057	.6409	2.7338	.3069	.8117	1.3592
Constant	-.7908	.3213		-.3150	.2657		.0290	.3001	

Note: [a] < .10; [b] < .05; [c] < .01; [d] < .001

use. For year 3, males, a cigarette smoking adult in the home, and transience in terms of the family residence changing more than one time in the last three years tended to be associated with students who reported more marijuana use. However, results of multivariate logistic regression revealed that more marijuana use was only significantly related to males and a cigarette smoking adult in the home at year 1; significantly to males and marginally significant to poor parent-child communication at year 2; and significantly to males.

Logistic regression based on level 2 variables provided strong support for the notion that familial variables can act as risk or protective factors for marijuana use in adolescents. These findings lend support to efforts to explore the effects of familial variables on the change rate of marijuana use in this population.

Growth Curve Model

A series of nested growth curve models were estimated to examine the change rate of marijuana use over the three years and the effects of familial factors on change rates of marijuana use (see Table 3). Overall, there was a consistent random effect associated with the mean rate of marijuana use at each time point suggesting that the pattern of use or non-use of marijuana was different among all students across three years.

Model 1 revealed a significant individual effect across three time points. The fully unconditional model suggested significantly different individual patterns of marijuana use change (intraclass correlation = .19). This effect prompted further modeling of the individual effect with the average rate of

Table 3. Random-Effects Regression on Marijuana Use

	Model 1		Model 2		Model 3		Model 4		Model 5	
Fixed Effects	MLE	SE	MLE	SE	MLE	SE	MLE	SE	MLE	SE
Constant, G_{00}	-0.482***	0.106	-0.859***	0.154	-0.942***	0.171	-1.344**	0.415	-0.825	0.525
Parental marital status, G_{01}							0.446	0.284	0.019	0.389
Student gender, G_{02}							-0.849***	0.223	-1.945**	0.707
Child-parent communication, G_{03}							0.224	0.224	0.210	0.225
Smoking adult at home, G_{04}							0.489*	0.232	0.502*	0.233
Moving residence, G_{05}							0.099	0.291	0.045	0.294
Gender x Marital status, G_{06}									0.919	0.564
Year (mean change rate), G_{10}			0.380***	0.106	0.760*	0.351	0.419***	0.109	0.420***	0.110
Year squared (acceleration rate), G_{20}	1.280***	1.132					-0.189	0.166		
Random effect										
Constant, U_0			1.450***	1.204	1.461***	1.209	1.398***	1.182	1.405***	1.186

Note: * $p < .05$; ** $p < .01$; *** $p < .001$

change of marijuana use in Model 2. It was found there was a significant, positive time effect on change of marijuana use (i.e., marijuana use increased across time among the students). Model 3 tested quadratic time effect on the change rate of marijuana use. However, its effect was not significant. Model 4 included level 2 covariates and consisted of poor parent-child communication, transience of the family, divorce of parents, a cigarette smoking adult in the home, and student's gender. Being male and having a cigarette smoking adult in the home significantly predicted use of marijuana at year 1. Student's gender and parental marriage status interaction terms also marginally related to marijuana use at year 1.

Discussion

Findings of the present investigation were based on a longitudinal study of Mexican American students and their parents over a three-year period. It provided important developmental understanding of marijuana use in this adolescent group. Overall there was a positive linear trend of increasing marijuana use across the three time points over the years. This is a trend seen with national studies in which the prevalence of substance use increases proportionately to the age/grade level of pre-adolescent and adolescent subjects (see for example, Johntson, O'Malley, & Bachman, 1996). However, different patterns of use or non-use of marijuana among all students across the three years was an important finding which may imply the emerging negative quadratic trend of marijuana use although it was not significant in Model 4. This may be related to the experimental use of marijuana during adolescence.

Both cross-sectional logistic regression and growth curve models consistently found students' gender an important predictor of marijuana use even after controlling for familial factors. In the present study males reported more marijuana use than females. The majority of national and state surveys of adolescent substance use find that males engage in illicit substance use at an earlier age and more often compared to their female counterparts (Gilbert & Cervantes, 1986; Murray, Perry, O'Connell, & Schmid, 1987; Newcomb, Maddahian, Skager, & Bentler, 1987; United States Department of Health and Human

Services, 1992). Based on this and other literature (Newcomb & Bentler, 1989; Stein et al, 1987), among males there are a myriad of risk factors that are associated with substance use including peer pressure, social deviance, emotional problems, and issues with self esteem, all of which tend to play a role particularly for males' relatively high substance use when compared to females. However, smoking among females is increasing at least in part because advertisers have targeted them as a highly lucrative market. Many of the studies cited note that females are catching up to, and in some cases becoming more frequent users of cigarettes when compared to males.

In one study conducted by Gfrorer and De La Rosa (1993), Latino female adolescents were found to engage in more illicit substance use than male adolescents. The researchers found that these females were more likely to report using illicit substances, including marijuana, as compared to males. This use of marijuana by Latino females is supported by the findings of the National Institute on Drug Abuse (U. S. Department of Health and Human Services, 1990; 1991) in which a trend over the last several years shows slightly higher rates of lifetime marijuana use among Latino females age 11-17 than among Latino males of the same age.

Having a cigarette smoking adult in the home was also shown to be a reliable predictor of the prevalence of marijuana use across the three years in the present study. This is consonant with the findings of the study by Gfroerer and De La Rosa (1993) in which 223 parent-child pairs were interviewed pertaining to family variables and children's' use of substances. An important finding of this study is that frequency of marijuana use by Latino youth was strongly related to the use of cigarettes by their mother. As Gfroerer and De La Rosa (1993) report, their data are supportive of the importance of children's modeling of parents' drug use behavior found in studies conducted with white and non-Latino families and their children (Brook, Whiteman, Gordon, Nomura, & Brook, 1968; Brooks, Whiteman, Nomura, Gordon, & Corton, 1988; Gfroerer, 1987). This study seems to support other research in that the impact of a cigarette smoking adult in the home has a tremendous influence on Latino children's behavior.

Transience, or relatively high rates of family residential mobility was found to be associated with higher rates of marijuana use by the Mexican American adolescents in the present study. Transience of the family in terms of the number of moves from one residence to another has been associated with a range of school related, psychological, and substance use issues (Humke & Schaefer, 1995). Studies found that family transience leads to depression, anxiety, and impacts overall life satisfaction for adolescents (Puskar & Ladely, 1992). Researchers theorize that family mobility impacts interpersonal relationships, overall adjustment, social and educational situations, and academic achievement to such an extent that transience must now be considered a constellation risk factor for substance use within this population. Perhaps multiple residential moves is associated with instability, uncertainty and a higher level of overall anxiety for Mexican American adolescents whether or not the move is precipitated by pleasant or not so pleasant circumstances. What we do know is that this is an issue that needs further study in this population in relation to substance use.

The impact of separation and divorce on adolescent development is influenced by a variety of factors, including when the divorce occurs, the nature and length of the family conflicts that lead up to and follow the divorce, the quality of the child's relationship with both the absent parent and the parent who have primary physical custody, and the economic circumstances of the family after the divorce. It is difficult to predict with great accuracy who will be severely affected by divorce. As a rule, boys appear to be more negatively influenced by divorce than are girls (Emery, Hetherington, & DiLalla, 1984). Girls who are affected often exhibit behaviors associated with anxiety and withdrawal.

Several studies have found that adolescents from intact homes (i.e., two natural parents reside in the home) are less likely to use marijuana and tend to use less frequently than adolescents from non-intact homes (e.g., single parent or stepparent homes; Hoffman, 1994; Wallace & Bachman, 1991; Wells & Rankin, 1991; Needle, Su, & Doherty, 1990; Flewelling & Bauman, 1990; Selnow, 1987; Smith & Paternoster, 1987) where parent monitoring and bi-gender role models exist. For example, Mednick, Baker, and Carothers (1990) found that parental divorce during adolescents leads to a significant increase in the probability of delinquency and adult criminality, as well as substance use. In the present study females were more negatively impacted than males by divorce in the home as evidenced by their use of marijuana. Perhaps the lack of a consistent male role model in the home, compounded with the intricacies and dynamics of interactions between daughters and their divorced mothers within the Mexican American culture increases the daughters' risk of substance use. Susceptibility to peer influence, vulnerability to poor personal decisions, and a strong desire to be accepted, or just "fit-in" with a substance using group may play significant roles in this finding.

Finally, problems in communication between parents and their children were found to have a significant impact on the use of marijuana for the subjects in this study. Parents whose communication style could be described as open in terms of the quality and quantity of verbal and non-verbal exchange (Barnes & Olson, 1982) had children who reported significantly less use of marijuana over the course of this study. Communication is generally viewed as one of the most crucial facets of interpersonal relationships. Further evidence of the belief that good communication skills are crucial to satisfaction with family relationships is offered by a large number of researchers (See Barnes & Olson, 1982 for a review). Barnes, Farrell, and Banerjee (1994, p. 197) concluded ". . . the quality of parenting [specifically in terms of communication] is critically important for adolescent outcome regardless of race or other sociodemographic characteristics" Positive adolescent-parent communication is a key element in the prevention of alcohol abuse and other deviant behaviors.

The impact of parental interactions via communication style becomes even more influential (as a risk or protective factor for substance use) in the context of "la familia" and the children within the Mexican American cultural traditions (Vasquez, 1998). Mexican American children in the present study who experience their parents' communication style as open, positive and supportive seem to mediate this relationship as a buffer or cushion against other environmental and familial risk factors for substance use.

Conclusion

Given the importance of familial factors in the Mexican American culture, it seems plausible to assume that substance use research efforts directed toward Mexican American adolescents would benefit from incorporating family and parents in the investigation. In the area of prevention (Kaufman, 1986; Kaufman & Borders, 1988; Faufman & Kaufmann, Stanton & Todd, 1982) researchers have strongly recommended family focused prevention interventions for drug abuse based on the effectiveness of controlled studies. A few programs have been developed for Latino families and researched (see Szapocznik, et al, 1989; Cervantes, 1993 for details). Barrett, Simpson, and Lehman (1988) found that a reduction in drug and alcohol use was related to family support among Mexican American youth in their first 3 months in drug intervention programs. However, there continues to be a dearth of substance use research in the area of family factors and Mexican American youth.

Whatever prevention and/or intervention orientation is espoused in the schools and community as a working model for Mexican American youth, the following components might well be considered for integration in programs as effective practices:

Focus on the family. Prevention/intervention efforts should focus on the importance of parent-child communication styles. An abundance of literature suggests that many Latino students are distinguished by a sense of loyalty to the family. Children from Latino homes are brought up with the notion that to bear the family name is a very important responsibility, and that their behavior reflects on the honor of the family. This cultural value stands in stark contrast to the "rugged individualism" that characterizes mainstream American values (Vasquez, 1998).

Parents and extended family members might benefit from intensive and extensive intervention efforts based within local schools but community led, focusing on modeling, and discussing more effective styles of communication compared to less effective styles of communication. An example of a culturally relevant, systems oriented approach to intervention efforts among Latino families is the Family Effectiveness Training for Latino Families (FET) (Szapocznik et al, 1986). In addition, parents must be made to understand the impact of their behavior upon their childrens' behavior (i.e., cigarette smoking), and the impact of divorce and transience upon individual children.

Focus on the student. Prevention/intervention efforts have traditionally focused exclusively on the student and tended to be adult centered. Contemporary models might consider the impact of social influences on adolescent alcohol and drug use. For example, Project SMART, a peer-led social influence prevention program, has been shown to be effective in delaying the onset of tobacco, alcohol, and marijuana use in a cohort of adolescents (Perry, 1996). Intensive and extensive substance use intervention training for school children, over multiple years is called for due to the growth curve effect discussed previously in this article.

In addition, Guthrie, Caldwell, and Hunter (1997) believe that health-promotion interventions in the next millennium must consider how gender socialization mediates the interaction of social class, ethnicity, and environment with self-efficacy, which in turn influences behavioral outcomes related to physical and mental health. Gender socialization is the process by which children learn how to think and act as boys or girls in a variety of situations. This process may be facilitated by environmental factors that provide reinforcement of specific gendered behaviors. An extension of this idea may mean the need for some gender segregation in our prevention/intervention efforts given the fact that females tend to respond differentially in the area of marijuana use compared to males when divorce of the parents occurs.

Focus on the teacher. Prevention/intervention efforts have not sufficiently sensitized and educated mainstream teachers to the intricacies of the Mexican American culture. For example, Vasquez (1998, p. 2) illustrates this point with the following, "Our attempts to reinforce youth must be based on values the student holds, and these often differ depending on the ethnic and social class background of the student. It is for this reason that teachers who comment that they 'treat all students the same' are not showing their democratic disposition, but rather that they are not yet prepared to teach in the pluralistic classrooms of American schools. Already more that one in every four students is an ethnic minority." All teachers, but particularly those that will implement intervention programs need to be specifically educated in the area of Distinctive Traits for Latino Students (see the Prevention Researcher, 1998).

Until comprehensive and multilevel prevention intervention efforts are constructed that address at least some of the concepts detailed in this article, movement toward a more substance-free, younger Latino generation may be further in the future than we would hope. Our effects need to be redoubled in the coming years in order to prevent the loss of an important ethnic generation for our nation.

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Endnote

Two additional analysis was conducted by coding marijuana use as none for those reporting no use or 1-2 times of use and as marijuana use for those reporting 3 or more times of use in last year. The purpose of this exercise was to explore the impact of "experimental marijuana use". Results of analysis were however similar to those reported in this paper.

Support for this research was provided by the National Institute of Drug Abuse Grant # R24DA07234 awarded to Jesse T. Zapata, David S. Katims, and Zenong Yin. The authors wish to thank the administration, staff, students, and teachers of Harlandale Independent School District, San Antonio, Texas. We also like to thank our research assistants at the Hispanic Research Center for their assistance in this research project.

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Multiple Linear Regression Viewpoints (ISSN 0195-7171) is published by the
AERA Special Interest Group on Multiple Linear Regression: General Linear Model
through the University of Alabama at Birmingham.